# REGULARIZATION OF HAMILTON'S PRINCIPLE FOR HIGHER-ORDER LAGRANGIAN SYSTEMS

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Let $L\colon T^{(k)}Q \to \mathbb{R}$ be a Lagrangian function defined on the $k$th order tangent bundle of the configuration manifold $Q$. A trajectory of the corresponding unconstrained higher-order Lagrangian system is a $C^k$ curve $q\colon [0,h] \to Q$ such that the action $\int_0^h L\left(q(t), \dot{q}(t), \dots, q^{(k)}(t)\right) dt$ has a critical point at $q$ with respect to variations arising from deformations of $q$ with fixed endpoint values for $q$ and its first $k-1$ derivatives. For $h = 0$, the constrained problem becomes nonregular since the constraint $q \mapsto \left(q(0), \dot{q}(0), \dots, q^{(k-1)}(0); q(h), \dot{q}(h), \dots, q^{(k-1)}(h)\right)$ is not a submersion. For a regular Lagrangian, meaning that $\left(\partial^2 L/\partial q^{(k)2}\right)$ is a regular matrix, we use a procedure similar to that given by G. Patrick for first-order systems (*Lagrangian mechanics without ordinary differential equations*, Rep. Math. Phys. 57, no. 3, 437–443, 2006) to regularize the variational principle at $h = 0$. From this we obtain the existence and uniqueness of solutions for small enough, positive $h$. This is important, for instance, when studying geometric integrators for higher-order systems, since it follows that the exact discrete Lagrangian is well-defined.