Ramsey Theory

Simon Griffiths
University of Oxford and PUC-RIO

Ramsey Theory is the study of inevitable substructures in large (usually discrete) objects. For example, consider colouring the edges of the complete graph K_n with two colours. In 1930, Ramsey [13] proved that if n is large enough, then we can find either a red complete subgraph on k vertices or a blue complete subgraph on ℓ vertices. We write $R(k,\ell)$ for the smallest such n. Another famous example is Van der Waerden's Theorem [16], that every r-colouring of the integers contains a monochromatic k-term arithmetic progression.

The course will begin with an overview of well known classical results in the theory, including the two theorems stated above. We will then go on to more advanced topics, including some (or all) of the the following:

- The recent new upper bound for diagonal Ramsey numbers $R(k,k)$ of Conlon [5].
- Bounds on the off-diagonal Ramsey numbers $R(3,k)$, proved in [1, 3, 9, 11, 15].
- Bounds on Ramsey numbers for small cliques versus large sparse graphs [2, 4, 6, 10, 12].

I will also discuss a large number of open problems.

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