

# Characterizations of convex approximate subdifferential calculus in Banach spaces\*

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## Abstract

We establish subdifferential calculus rules for the sum of convex functions defined on normed spaces. This is achieved by means of a condition relying on the continuity behaviour of the inf-convolution of their corresponding conjugates, with respect to any given topology intermediate between the norm and the weak\* topologies on the dual space. Such a condition turns out to be also necessary in Banach spaces. These results extend both the classical formulas by Hiriart Urruty-Phelps [17] and by Thibault [27].

**Key words.** Convex functions, approximate subdifferential, calculus rules, approximate variational principle.

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