The Hilbert scheme of 4 points in \mathbb{P}^3

Adriana Rodrigues da Silva - UFU

Abstract: Let $\operatorname{Hilb}^r \mathbb{P}^n$ the Hilbert scheme that parametrizes the family subschemes of dimension 0 and length r in \mathbb{P}^n . Our interest is to describe geometrically $\operatorname{Hilb}^4 \mathbb{P}^3$, configurations of 4 points in \mathbb{P}^3 .

Four points in general position determine the unordered vertices of a tetrahedron. We can choose two vertices and draw the support line. Automatically, the two remaining vertices are contained in another line.

We have six choices for the first pair of vertices.

Looking backwards, first we take two lines and set up two points in each.

The ideal of a general configuration of four points in \mathbb{P}^3 is defined by a system of 31 quartic, obtained by the product of equations that define each point.

Settings in a special position to the same account returns a subspace of the quartic smaller dimension, the problem is to extend the bundle of 31 quartic.

To do this, we use the Fitting ideals, which control the centers of successive blow up. Thus, we construct a covering generally six to one for the Hilbert scheme of 4 points in \mathbb{P}^3 .