Conditional Weighted Expected Shortfall, Conditional Distortion Risk Measures, and Application to Risk Capital Allocation

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Abstract :
Based on the concepts of measurable upper envelopes and conditional lower quantiles, we define conditional distortion risk measures via a stochastic integral representation. We show various properties of conditional distortion risk measures, including coherence with respect to distortion processes with concave paths. Conditional weighted expected shortfall arises as a special case of conditional distortion risk measures. We also give a definition via an explicit density on a modelling setup with stochastic levels, involving generalised conditional expectations based on sigma-integrability.

Then we prove several properties and give several alternative representations of conditional (weighted) expected shortfall. Furthermore, we point out the link to dynamic risk measures and show a supermartingale property.

In the next step we introduce contributions to conditional weighted expected shortfall and prove several properties. In particular, it is possible to derive the contribution of a subportfolio to the whole portfolio in order to be able to identify main risks. Conditional weighted expected shortfall includes beta- and alpha-value-at-risk as special cases. We end with some applications including a time series example. (Based on joint work with Karin Hirhager and Jonas Hirz.)