

## Valuation in illiquid markets and the Feynman-Kac representation

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After a long period with an abundance of liquidity in the markets, the 2007 – 2009 financial crisis illustrated in a dramatic way how fundamental liquidity risk is. In this situation many securities with an excellent rating could no longer be traded. What is the value of the instruments under these market conditions? The classical valuation theory which is based on the law of one price assumes implicitly that market participants can trade freely in both directions at the same price. In the absence of perfect liquidity the law of one price should be replaced by a two price theory where the terms of trade depend on the direction of the trade.

We develop here a static as well as a continuous time theory for two price economies. The two prices are termed bid and ask or lower and upper price but they should not be confused with the vast literature relating bid-ask spreads to transaction costs or other frictions involved in modeling financial markets. The bid price arises as the infimum of test valuations given by certain market scenarios whereas the ask price is the supremum of such valuations. The two prices correspond to nonlinear expectation operators. The approach is made operational by using probability as well as measure distortions. We discuss in detail the validity of the Feynman-Kac representation of solutions of partial integro-differential equations on which the dynamic theory is based.

Specific models which are driven by purely discontinuous Lévy processes are considered. The approach is illustrated to price contracts with extremely long maturities. We also discuss the valuation of insurance loss liabilities modeled via compound Poisson processes.

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### References

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