Construction of a minimal mass blow up solution of the modified Benjamin-Ono equation

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This talk is based on a joint work with Yvan Martel (Ecole Polytechnique)

Abstract: We construct a minimal mass blow up solution of the modified Benjamin-Ono equation (mBO), which is a classical one dimensional nonlinear dispersive model.

Let $Q \in H^{\frac{1}{2}}$, $Q > 0$, be the unique ground state solution associated to mBO. We show the existence of a solution S of mBO satisfying $\|S\|_{L^2} = \|Q\|_{L^2}$ and

$$S(t) - \frac{1}{\lambda^{\frac{1}{2}}(t)} Q\left(\frac{\cdot - x(t)}{\lambda(t)}\right) \to 0 \quad \text{in} \;\; H^{\frac{1}{2}}(\mathbb{R}) \text{ as } \;\; t \downarrow 0,$$

where

$$\lambda(t) \sim t, \quad x(t) \sim -|\ln t| \quad \text{and} \quad \|S(t)\|_{\dot{H}^{\frac{1}{2}}} \sim t^{-\frac{1}{2}} \|Q\|_{\dot{H}^{\frac{1}{2}}} \quad \text{as } t \downarrow 0.$$

This existence result is analogous to the one obtained by Martel, Merle and Raphaël (J. Eur. Math. Soc., 17 (2015)) for the mass critical generalized Korteweg-de Vries equation (gKdV). However, in contrast with the gKdV equation, for which the blow up problem is now well-understood in a neighborhood of the ground state, S is the first example of blow up solution for mBO.

The proof involves the construction of a blow up profile, energy estimates as well as refined localization arguments, developed in the context of Benjamin-Ono type equations by Kenig, Martel and Robbiano (Ann. Inst. H. Poincar, Anal. Non Lin., 28 (2011)). Due to the lack of information on the mBO flow around the ground state, the energy estimates have to be considerably sharpened here.