Coherence and Correspondence Decision Criteria: How to Evaluate Processes

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The rationality of decisions is usually evaluated behaviorally, but sometimes there are good reasons to step back and evaluate agents' decision *processes* instead. For example, a decision process can be the common cause of many individual choices – useful for predicting future choices – and achieving pedagogical or "meliorative" goals may require knowledge of the processes agents actually use or might be taught (see e.g. [Kitcher, 1992]). Additionally, if agents rely on simple heuristics to make decisions, we may wish to know how rational those heuristics are ([Gigerenzer and Selten, 1999, Gigerenzer, 2000, Gigerenzer et al., 2011]). How decision processes should / could be evaluated is an open question, however, and here I explore an answer (that I argue for elsewhere on theoretical grounds) [Rich, ms.] using lottery choice as a test case. I argue that even when evaluating decision processes rather than observed choices, an axiomatic coherence standard is the only viable option; specifically, processes should be evaluated by simulating their performance over a range of cases and determining how often they conform to the relevant axioms.

I first apply this procedure to evaluate and compare several lottery choice heuristics, specifically Minimax, Maximax, the Hurwicz Criterion with five different alpha values (see [Luce and Raiffa, 1957, Ch. 13]), and the Priority Heuristic [Brandstätter et al., 2006], as well as the simple maximization of expected value (EV). The Priority Heuristic is of special interest because its creators argue that it is our best explanation for people's lottery choices and it predicts many empirical decision phenomena (e.g. the Allais Paradox [Allais, 1953]) in which people violate Expected Utility Theory. I simulate each process' choices on pairs of lotteries sampled widely from the decision science literature (29070 choices for each process). Testing each process for violations of the expected utility axioms, I find that all the heuristics except Maximax do on rare occasion violate independence, and the Priority Heuristic stands out in additionally producing a fairly large number of transitivity violations (12% of its opportunities), though these violations tend to involve a highly implausible choice. The prima facie interpretation of these results is that maximizing EV and maximaxing are perfectly rational, Minimax and the Hurwicz Criterion are quite rational but imperfect, and the Priority Heuristic is the least rational but still reasonably good (i.e. it should rarely result in violations in practice).

A possible objection to this evaluation procedure comes from Berg, who criticizes coherence as the measure of rationality; he suggests that the use of money-pump arguments implies that wealth (rather than coherence) is of fundamental value [Berg, 2014] and gives examples in which violating classical rationality norms makes individuals better off (e.g. [Berg and Lien, 2003]). The first response to this objection is to note that Expected Utility Theory – rather than EV maximization – is used as the rationality standard for situations of risk because objective ("correspondence") standards neglect the legitimate role played by subjective risk preferences (recall the St. Petersburg Paradox, for example [Bernoulli, 1954]). Despite this, I agree that the coherence metric would have something to answer for if there were really no relationship, or an inverse relationship, between coherence and more objective metrics that agents seem to endorse, as Berg suggests might be the case.

For this reason, the second part of my analysis compares the decision processes according to choice EV and looks for relationships between axiom violations and diminished EV. In order to have a large enough sample of independence violations, I generate new lotteries such that a heuristic's choice between a pair of lotteries in the original set implies particular choices for 25 new pairs; given this increased opportunity, the Priority Heuristic does violate independence much more frequently for particular kinds of new pairs. Both transitivity and independence violations are strongly associated with diminished EV for the Priority Heuristic's choices, but so is the magnitude of the payoffs. Minimax also tends to have far lower EV than the others, and again especially as payoffs increase. Yet this is exactly what we would expect if agents maximize expected utility but have very concave utility functions. The Priority Heuristic has lower EV than the rest of the heuristics, but the difference is much less extreme than between Minimax and any other process and it is not as readily explained by risk aversion.

In summary, this analysis shows that the disadvantages people realize from using inferior processes are likely to be modest (i.e. the degree of irrationality will not in general be large); still, when processes can be identified, process analysis may be more informative than ordinary expected utility analysis because it provides a way of quantifying an agent's degree of rationality. The coherence standard of Expected Utility Theory is also vindicated in two ways: First, we cannot draw strong conclusions from EV analysis for the reason cited above, and no other natural correspondence metric is available to enable us to do so. Second, the EV analysis supports cautious optimism that coherence is correlated with objective success in the domain of lottery choice. Expected Utility Theory therefore provides a solution to the problem of evaluating the rationality of decision processes for situations of risk.

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