Renormalized-generalized solutions for the KPZ equation

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The KPZ equation describes the evolution of the profile of the interface, $h(x,t)$, at position x and time t:

$$\begin{cases} \partial_t h(t,x) = \triangle h(t,x) + (\partial_x h(t,x))^2 + W(t,x) \\ h(0,x) = f(x) \end{cases} \tag{1}$$

where $W(t,x)$ is a space-time white noise.
From a mathematical point of view, equation (1) is ill-posed since the solutions are expected to look locally like a Brownian motion. The term $(\nabla h(t,x))^2$ cannot make sense in a classical way.
L. Bertini and G. Giacomin in [BG], proposed that the correct solutions for the KPZ equation are obtained by taking the logarithm of a solution of the stochastic heat equation with multiplicative noise. This is known as the Cole-Hopf solution for the KPZ equation, the evidence for this solution is overwhelming even though it satisfies, at a purely formal level, the equation

$$\begin{cases} \partial_t h(t,x) = \triangle h(t,x) + (\partial_x h(t,x))^2 + W(t,x) - \infty \\ h(0,x) = f(x) \end{cases} \tag{2}$$

where ∞ denotes an infinite constant.
The above remark indicates that a key problem is to find an appropriate notion of solution for the KPZ equation such that incorporates the Cole-Hopf solution. We construct spaces of generalized stochastic processes that can be used to renormalize the divergent term appearing in (2). Moreover, we prove that the Cole-Hopf solution becomes then a well defined solution to the KPZ equation. At the level of sequences, our space of generalized stochastic processes looks like a Colombeau space and at the level of Schwartz distributions it looks like a generalized stochastic processes space in the sense of Itô-Gelfand-Vilenkin.

References

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- [KPZ] M. Kardar, G. Parisi, and Y. Zhang, *Dynamic scaling of growing interfaces*, Phys. Rev. Lett. 56 (1986) 889-892.