CRITICAL SOLUTIONS OF NONLINEAR EQUATIONS

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Resumo/Abstract:
It is known that when the set of Lagrange multipliers associated with a stationary point of a constrained optimization problem is not a singleton, this set may contain so-called critical multipliers. This special subset of Lagrange multipliers defines, to a great extent, the stability pattern of solution in question subject to parametric perturbations. Criticality of a Lagrange multiplier can be equivalently characterized by the absence of the local Lipschitzian error bound in terms of the natural residual of the optimality system. In this work, taking the view of criticality as that associated to the error bound, we extend the concept to general nonlinear equations (not necessarily with primal-dual optimality structure). Among other things, we show that while degenerate noncritical solutions of nonlinear equations can be expected to be stable only subject to some poor "asymptotically thin" classes of perturbations, critical solutions can be stable under rich classes of perturbations. This fact is quite remarkable, considering that in the case of nonisolated solutions, critical solutions usually form a thin subset within all the solutions. We also note that the results for general equations lead to some new insights into the properties of critical Lagrange multipliers (i.e., solutions of equations with primal-dual structure).