

**Michael Boshernitzan.** Scales of functions and applications in ergodic theory and number theory.

**Abstract.** G.H. Hardy in his book “Orders of Infinity” introduced a class  $L$  of real functions defined in a neighborhood of  $+\infty$  by means of certain formula involving the variable  $x$ , real constants, algebraic operations and the functional symbols  $\exp()$  and  $\log()$ . He proved that this class forms a scale: it is linearly ordered by eventual dominance at  $+\infty$ .

We describe some abstract definitions which lead to various classes of scales, some much larger than  $L$ . The functions in these scales satisfy various differential, difference and functional equations. We present some known results on the structure of these classes and state a number of open questions and conjectures. Some applications to the questions on uniform distribution in ergodic theory and number theory will be sketched, as well as the connection to the 16th Hilbert problem.

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