Aspects of branching Brownian motion

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The one-dimensional branching Brownian motion is one of the simplest example of branching particle systems. In its most elementary form, the model can be described as follows: particles move on the real line according to independent Brownian motions and live for independent exponential lifetimes. Upon dying they are replaced by exactly two offspring particles at the same position who then start to follow the same dynamic, and so on.

Remarkably, this model and its variants are intimately connected, in a way similar to the relation between the heat equation and the standard Brownian motion, to the celebrated F-KPP equation (introduced in 1937 simultaneously by Fisher and Kolmogorov, Petrovskii and Piskunov) which is a partial differential equation describing various reaction-diffusion phenomena which can give rise to front propagation. It is a central feature of various models related to combustion, chemistry, biology and ecology (indeed, Fisher's original goal was to study how an advantageous allele spreads in a geographically distributed population). It is one of the simplest examples of a PDE that admits traveling wave solutions and which to a large extent started the field of semi-linear parabolic PDEs.

This connection, first studied by McKean in 1975, is at the heart of the famous result of Bramson (1983). The interplay between the F-KPP equation and branching random walks was further exploited in several directions, both to study branching walks and the F-KPP equation itself. More recently, Brunet, Derrida and their collaborators completely regenerated the field by making numerous conjectures coming from physics that concern a stochastic version of the F-KPP equation (with added noise), certain branching random walk with {\it selection} or {\it absorption} and the front of branching particle systems and their relations to the original PDE.

In these lectures I will explore several aspects of the branching Brownian motion which are related to this connection focusing on a ``modern'' approach {\it via} spinal decomposition techniques. The main recent developments I wand to focus on will be

1) the extremal point process of the branching Brownian motion,
2) the study of branching brownian motion with absorption,
3) the genealogy of populations under selection.
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