

Nonnegativity certificates on real algebraic curves.

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Let $X \subseteq \mathbb{P}^n$ be an irreducible, non-degenerate and totally real algebraic curve. Let F be a form of degree $2s$ in its real homogeneous coordinate ring $R_X := \mathbb{R}[x_0, \dots, x_n]/I(X)$.

We prove that if F is non-negative on $X(\mathbb{R})$ and the integer k satisfies

$$k \geq \max \left(\deg(X) - n + 1, \left\lfloor \frac{2p_a(X) - 1}{\deg(X)} \right\rfloor + 1 \right)$$

then there exist sum of squares q_1 and q_2 in R_X of degrees $2k$ and $2(k + s)$ such that $pq_1 = q_2$. Moreover, we prove that the above bound on k is (asymptotically) sharp.

Our results give new exact algorithms for polynomial optimization over real algebraic curves via semidefinite programming. Moreover, we obtain the only currently known sharp bounds for algebraic certificates of non-negativity on positive-dimensional varieties. These results are ongoing joint work with G. Blekherman and G.G. Smith.