

New Trends in Onedimensional Dynamics

Celebrating the 70th anniversary of Welington de Melo

Rio de Janeiro, November 14 - 18, 2016

Title: Mating quadratic maps with the modular group

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Abstract: In 1994 S. Bullett and C. Penrose introduced the 1 complex parameter family of $(2 : 2)$ holomorphic correspondences $_a$:

$$\left(\frac{aw-1}{w-1}\right)^2 + \left(\frac{aw-1}{w-1}\right)\left(\frac{az+1}{z+1}\right) + \left(\frac{az+1}{z+1}\right)^2 = 3$$

and proved that for every value of $a \in [4, 7] \subset \mathbb{C}$ the correspondence $_a$ is a mating between a quadratic polynomial $Q_c(z) = z^2 + c$, $c \in \mathbb{C}$ and the modular group $PSL(2, \mathbb{C})$. They conjectured that this was the case for every member of the family $_a$ which has a in the connectedness locus.

For every member of the family $_a$, the branch of $_a$ which fixes 0 is parabolic, with multiplier at the parabolic fixed point equal to 1. This fact suggests that the family $_a$ is deeply different from the family of quadratic polynomials, and it also suggests that the optimum description of the correspondences $_a$ might be as matings between the modular group and members of some family of parabolic quadratic maps.

This is indeed the case: in a joint work with S. Bullett we prove that every member of the family $_a$ which has a in the connectedness locus is a mating between the modular group and an element of the parabolic quadratic family $Per_1(1)$.