Algorithms for the Nash Equilibrium Problem

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Abstract:

We discuss in this work algorithms for the Nash Equilibrium Problem - NEP, which can be treated as a particular case of general Equilibrium Problem.

Iusem and Nasri [1] introduce Augemented Lagrangian methods for solving finite dimensional equilibrium problems whose feasible sets are defined by convex inequalities, generalizing the proximal Augemented Lagrangian method for constrained optimization.

Based on this work we present two metodologies to solve the NEP. In the smooth case, namely the functions involved in the problem are differentiable, we have to solve in each iteration of the methods a nonlinear and not differenciable system $G(x) = 0$, where $G : \mathbb{R}^n \to \mathbb{R}^n$. Both metodologies consist in a smoothing of the not differentiable term of G forming a new differentiable system $\bar{G}(x) = 0$. The first metodology use the Newton method to solve $\bar{G}(x) = 0$. We show that the Newton direction is well defined and the convergence results shown by Iusem and Nasri are still valid. The second metodology consists in using subgradient methods applied to problem $min\|\bar{G}(x)\|^2$ instead of solve directly $\bar{G}(x) = 0$.

To show the efficiency of the algorithms we have done numerical tests with known problems from de literature. Then we compare our results with results of other known methods.

References

[1] Iusem, A. N. and Nasri, M. Augmented Lagrangian Methods for Equilibrium Problems. *RAIRO. Recherche Opérationnelle*, 44:5–26, 2010.