Preservation of Volumes in Nonholonomic Mechanics

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The equations of motion of a mechanical system subjected to nonholonomic linear constraints can be formulated in terms of a linear almost Poisson structure in a vector bundle. We study the existence of invariant measures for the system in terms of the unimodularity of this structure. In the presence of symmetries, our approach allows us to give necessary and sufficient conditions for the existence of an invariant volume, that unify and improve results existing in the literature [1, 5, 8, 11]. We present an algorithm to study the existence of a smooth invariant volume for nonholonomic mechanical systems with symmetry and we apply it to several concrete mechanical examples.

Our method allows us to prove non-existence of an invariant measure in general and distinguish the cases of its existence in the following known problems:

(i) As considered in [9, 10], the motion of a rigid body with a planar section that rolls without slipping over a fixed sphere.

(ii) The motion of an inhomogeneous sphere whose center of mass does not coincide with its geometric center that rolls without slipping on the plane (Chaplygin's top).

(iii) A homogeneous triaxial ellipsoid that rolls without slipping on the plane.

(iv) The rolling without slipping of an inhomogeneous sphere, whose center of mass coincides with its geometric center, over a circular cylinder.

During the talk I intend to explain how many important geometrical ideas developed by Jair Koiller and his collaborators in [2, 6, 7] and other references have been influential in our work.

The references to our work are [3, 4].

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