Flow of nonsmooth vector fields and applications. Part I

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Abstract:
At the beginning of the 1990→s, DiPerna and Lions made a deep study on the connection between transport equations and ordinary differential equations. In particular, by proving existence and uniqueness of bounded solutions for transport equations with Sobolev vector-fields, they obtained (roughly speaking) existence and uniqueness of solutions for ODEs for a.e. initial condition. Ten years later, Ambrosio extended this result to BV vector fields, providing also a new axiomatization of the theory of flows, more based on probabilistic tools.

In recent years, several new extensions have been obtained, that give rise to applications to PDEs which include some systems of conservation laws, semi-geostrophic equations, the linear Schrodinger equation and the Vlasov-Poisson equation. In the first part of the lectures (by L. Ambrosio), we will introduce the general theory of flows, covering the duality between the ODE well-posedness and the PDE well-posedness and presenting basic classes of vector fields (Sobolev, BV, ...) where this theory applies. In the second part of the lectures (by A. Figalli), we shall focus on the more recent extensions and their applications to PDEs.