Refined curve counting and tropical geometry

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Abstract:
The Severi degrees count the number of plane curves of degree d with delta nodes though d(d+3)/2-delta general points. One can study this problem more generally for a linear system on a general algebraic surface. For toric surfaces we introduce and study refined Severi degrees via tropical geometry. These are Laurent polynomials in a variable y which interpolate between Severi degrees and and totally real Welschinger invariants (with only real point conditions) in real algebraic geometry. We use tropical geometry to compute them in terms of an action of a Heisenberg algebra on a Fock space. More generally we use tropical geometry to define and study refined Broccoli invariants which interpolate between general Welschinger invariants (allowing also complex point conditions)and descendent Gromov-Witten invariants.