In this talk I will introduce a new distance between nonnegative finite Radon measures in R^d. The distance is constructed by a Lagrangian approach (dynamical minimization of an action functional), which is similar to the Benamou-Brenier formula. Compared to the classical setting of optimal transport for probability measures, the new distance has the advantage of allowing for mass variations, and the theory does not require any finite moments or decay at infinity. The structure allows to develop a Riemannian calculus /à la Otto./
Among other results, we obtain: completeness, local equivalence with other distances, lower semi-continuity, existence of geodesics, and characterization of Lipschitz curves.

If time permits I will discuss the application to a fitness-driven model of heterogeneous population dynamics: once suitably interpreted as a gradient flow we show exponential convergence to the unique steady state with explicit rate.