From particle systems to collisional kinetic equations

Laure Saint-Raymond (École Normale Supérieure - Paris)

Abstract:

1. The low density limit: formal derivation
Consider a deterministic system of N hard spheres of diameter $\eps$. Assume that they are initially independent and identically distributed. Then, in the limit when $N\to \infty$ and $\eps \to 0$ with $N\eps^2=1$ (Boltzmann-Grad scaling), the one-particle density can be approximated by the solution to the kinetic Boltzmann equation. In particular, particles remain asymptotically independent. In the first lecture, we will present the formal derivation of this low density limit, and discuss two important features, namely the propagation of chaos and the appearance of irreversibility.

2. A short time convergence result
Lanford's theorem states that in the Boltzmann-Grad limit the one-particle density converges to the solution of the kinetic Boltzmann equation almost everywhere on a short time interval (corresponding actually to a fraction of the average first collision time). The proof relies on a careful study of the recollision mechanism (which is not described by the Boltzmann dynamics), and on a priori bounds obtained by a Cauchy-Kowalewski argument.

In the second lecture, we will give a sketch of this proof, and show that the time restriction is due to the lack of global a priori bounds.