A $C^\infty$-closing lemma for three-dimensional Reeb flows via embedded contact homology

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Resumo/Abstract:
We will explain a proof of a $C^\infty$-closing lemma for Reeb flows on closed contact three-manifolds. As an immediate consequence, it follows that for any closed contact three-manifold with a $C^\infty$-generic contact form the union of all periodic Reeb orbits is dense. Our proof is based on recent developments in quantitative aspects of embedded contact homology (ECH). ECH is a kind of Floer homology defined for closed contact three-manifolds, which has been developed by M.Hutchings and his collaborators. In particular, a key fact is that the asymptotics of ECH spectral invariants recover the volume of a contact manifold, which was proved by D.Cristofaro-Gardiner, M.Hutchings and V.G.B.Ramos. A proof of a $C^\infty$-closing lemma for Hamiltonian diffeomorphisms of closed surfaces (joint work with Masayuki Asaoka) will be also sketched. The proof uses the above result for three-dimensional Reeb flows and analysis of an area-preserving map near its fixed point.