SOLVING A MIN-MAX CONTROL PROBLEM VIA AN ASSOCIATED DISCRETE PROBLEM

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ABSTRACT

In this paper, we study a min-max optimal control problem. We consider in the interval $[0, T]$ a dynamic system which evolves according to the ordinary differential equation

$$\begin{cases} \dfrac{dy}{ds}(s) = g(s, y(s), \alpha(s)) & 0 \leq s \leq T, \\ y(0) = x \in \Omega \subseteq \mathbb{R}^r, & \Omega \text{ an open domain} \end{cases}$$

where $g(s, y(s), \alpha(s)) = A(s)y(s) + B(s)\alpha(s) + C(s)$, A, B and C are Lipschitz continuous functions on $[0, T]$.

The optimal control problem consists in minimizing the functional $J : \Omega \times \mathcal{U} \mapsto \mathbb{R}$,

$$J(x, \alpha(\cdot)) = \operatorname{ess\,sup} \{f(y(s), \alpha(s)) : s \in [0, T)\},$$

over the set of controls

$$\mathcal{U} = \{\alpha : [0, T] \to U \subset \mathbb{R}^m : \alpha(\cdot) \text{ measurable}\}$$

where U is compact and convex, and f is a Lipschitz continuous function on $\Omega \times U$ and independent of α. The value function is

$$u(x) = \inf_{\alpha \in \mathcal{U}} J(x, \alpha).$$

Using classical convex analysis [1], we obtain a set of necessary optimality conditions that must be verified by any optimal control policy.

We propose an associated discrete time problem and we prove that these conditions are necessary and sufficient for this problem. Based on one of this optimality condition, we design an algorithm to solve the discrete problem. We prove that the algorithm converges, i.e., either the sequence constructed by the algorithm finishes at an optimal point or else it is infinite and every accumulation point is optimal. The algorithm gives us the optimal discrete control and the optimal value of the discrete problem.

We prove the linear convergence of the discrete value function to the continuous value function,

$$\left| u(x) - u^h(x) \right| \leq L\, h,$$

where $u^h(\cdot)$ is the discrete value function and h is the size of the parititon of the interval $[0, T]$.

We also prove that the optimal discrete control gives a good approximation of the value function in the continuous problem, in other words, if $\bar{\alpha}$ is an optimal control for the discrete problem, then

$$|J(x, \bar{\alpha}) - u(x)| \leq C\, h.$$

References

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