

# On Serrin's overdetermined problem and a conjecture of Berestycki, Caffarelli and Nirenberg

Jun-Cheng Wei

Univ. of British Columbia

## Resumo/Abstract:

In 1971, Serrin proved that the only bounded domain for which the overdetermined problem

$$\Delta u + f(u) = 0, u > 0 \text{ in } \Omega, u = 0 \text{ on } \partial\Omega, \partial_\nu u = C \text{ on } \partial\Omega$$

admits a solution is the ball. In 1997, Berestycki, Caffarelli and Nirenberg proposed the following conjecture for unbounded domains: If Serrin's problem has a solution and  $\Omega^c$  is connected, then either  $\Omega$  is a half space, or  $\Omega$  is the cylinder  $B \times R^{N-k}$ , or complement of the ball or cylinder. In this talk I will present several (positive and negative) results towards the Berestycki-Caffarelli-Nirenberg conjecture. In particular we show that in the case of epigraph  $\Omega = \{x_N > \varphi(x')\}$  the conjecture is always true in dimension 2, and true for  $3 \leq N \leq 8$  provided  $\frac{\partial u}{\partial \nu} > 0$ , and is false when  $N \geq 9$ . A key observation is the connection with the one-phase free boundary problem.