

On Serrin's overdetermined problem and a conjecture of Berestycki, Caffarelli and Nirenberg

Jun-Cheng Wei

Univ. of British Columbia

Resumo/Abstract:

In 1971, Serrin proved that the only bounded domain for which the overdetermined problem

$$\Delta u + f(u) = 0, u > 0 \text{ in } \Omega, u = 0 \text{ on } \partial\Omega, \partial_\nu u = C \text{ on } \partial\Omega$$

admits a solution is the ball. In 1997, Berestycki, Caffarelli and Nirenberg proposed the following conjecture for unbounded domains: If Serrin's problem has a solution and Ω^c is connected, then either Ω is a half space, or Ω is the cylinder $B \times R^{N-k}$, or complement of the ball or cylinder. In this talk I will present several (positive and negative) results towards the Berestycki-Caffarelli-Nirenberg conjecture. In particular we show that in the case of epigraph $\Omega = \{x_N > \varphi(x')\}$ the conjecture is always true in dimension 2, and true for $3 \leq N \leq 8$ provided $\frac{\partial u}{\partial \nu} > 0$, and is false when $N \geq 9$. A key observation is the connection with the one-phase free boundary problem.