Let $\Omega \subset \mathbb{R}^n$ be a bounded domain with regular boundary. Let $\alpha \in (0,1)$, and $\Gamma \subset \partial\Omega$ a window such that $|\Gamma|_{n-1} = \alpha|\Omega|_{n-1}$. The optimal Sobolev trace constant is defined as

$$S(\Gamma) := \inf_{v \in W^{1,p}_{\Gamma}(\Omega)} \frac{\int_{\Omega} |\nabla v|^p + |v|^p \, dx}{\int_{\partial\Omega} |v|^p \, dS},$$

where $W^{1,p}_{\Gamma}(\Omega)$ is the set of functions $v \in W^{1,p}(\Omega)$ such that $v|_{\Gamma} = 0$.

In [Del Pezzo, F. Bonder, Neves, JDE (2011)], the authors study the following problem: minimize $S(\Gamma)$ among all admissible windows, i.e.

$$S_{\alpha} = \inf_{\Gamma \in \Sigma_{\alpha}} S(\Gamma),$$

where $\Sigma_{\alpha} = \{\Gamma \subset \partial\Omega \colon \text{are measurable for}\, dS \text{ and } |\Gamma|_{n-1} = \alpha|\partial\Omega|_{n-1}\}$.

In the above mentioned work the authors show the existence of an optimal window Γ^*. Moreover it is shown that if u^* is the eigenfunction associated to $S(\Gamma^*)$ then $\{u^* = 0\} \cap \partial\Omega = \Gamma^*$.

In this work we study the behavior of this optimals windows when the domain Ω is perturbed periodically by a sequence of domains Ω_{ϵ}. Then we analyze the behavior of these optimals windows Γ^*_{ϵ} as $\epsilon \to 0$ and try to determine whether they approximate Γ^* in someone reasonable sense.

We find that the behavior of the trace constants $S_{\alpha,\epsilon}$ and the optimals windows Γ^*_{ϵ} depends strongly of the amplitude of the oscillations. We distinguish three cases: i.- Subcritical case: In this case the oscillations are very big and the trace constant converges to zero. ii.- Supercritical case: In this case the oscillations are very small and there are convergence to the unperturbed problem. iii.- Critical case: In this case the amplitude compensates with the oscillations and this is reflected in the appearance of a weight term.

The results presented here are new even in the linear eigenvalue problem that corresponds to $p = 2$.