Curvature of Real Algebraic Sets

Jean-Jacques Risler

1 Introduction

Let $X \subset \mathbf{C}^n$ which is real (invariant by complex conjugation), $X_{\mathbf{R}}$ its real part. Smith Theory implies the following Thom-Smith inequality between the sum of Betti numbers (mod 2) of $X_{\mathbf{R}}$ and X :

$$\sum_{i \geq 0} h_i(X_{\mathbf{R}}) \leq \sum_{i \geq 0} h_i(X)$$

We prove in several cases listed below a similiar inequality where the sum of Betti numbers is replaced by the total curvature.
The curvature function on X (resp. ($X_{\mathbf{R}}$) is the Lipschitz-Killing curvature (resp. the Gauss Curvature) which enter in the "Gauss-Bonnet Formula".

2 Affine Algebraic case

In this cases sharpness is proved in any degree d up to any $\varepsilon > 0$.

3 Cases of Amoebas

In this case sharpness is true only in the case of "Simple Harnack Curves" as defined by G.Mikhalkin.

4 Tropical Hypersurfaces

In this case, sharpness of the inequality is proved for any smooth tropical hypersurface (Joing work with Benoit Bertrand ans Lucia Lopez de Medrano).