Analytical issues for non-local multi-linear interaction models: The Boltzmann and related equations

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Abstract:
The Boltzmann equation models the evolution of continuum random processes for multi-linear dynamics. Nowadays, diverse models based on the ideas of Boltzmann and Maxwell, referred also as collisional kinetic transport in particle interacting systems, are widely used in modeling phenomena ranging from rarefied classical gas dynamics, inelastic interacting systems in granular or polymer kinetic flows, collisional plasmas and electron transport in nanostructures in mean field theories, to self-organized or social interacting dynamics. This type of models share a common description based in a Markovian framework of birth and death processes in a multi-linear setting.
Following the Introductory lectures "From particle systems to collisional kinetic equations" by Laure Saint-Raymond, in the first two lectures, we will focus on diverse analytical issues depending on the properties of the transition probability rates associated to the Markovian process. We will discuss both the space homogeneous as well as the inhomogeneous problems. The results strongly depend on the structure of transition probability rates, which controls regularity, high-energy tail properties, as well as long time behavior of the solutions to steady or self-similar states. We will also discuss the Coulomb potential limit case the yields the Landau equation widely use in collisional plasma theory.
The third lecture will focus on an interesting result that distinguish the characterization of space inhomogeneous stationary solutions of in all space vs. a tori, where the effects of dispersion and dissipation interplay producing unexpected effects. We analyze the existence and long time behavior of solutions of the space inhomogeneous Boltzmann equation in the whole space for initial data in the vicinity of a global Maxwellian, and show, surprisingly, the existence of a scattering regime that leads to the construction of eternal solutions that do not converge to such global Maxwellian, as expected from the H-theorem.