HARDY-SOBOLEV TYPE INEQUALTIES WITH MONOMIAL WEIGHTS

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Resumo/Abstract:
In this talk we ask ourself about the validity of the following family of inequalities

$$\left(\int_\Omega |u(x)|^q v(x)dx\right)^{1/q} \leq C\left(\int_\Omega |\nabla u(x)|^p w(x)dx\right)^{1/p}, \qquad (1)$$

where v and w are weights functions and Ω is some subset of $\mathbb{R}^N$, usually $\Omega = \mathbb{R}^N$, $\Omega = \mathbb{R}^N_+$ (the half-space), or $\Omega = (\mathbb{R}^N)_+$ (the positive cone).

There are some famous weights that have been known and vastly used for a long time, for instance $v(x) = w(x) = 1$ (Sobolev-Gagliardo-Nirenberg inequality), $v(x) = |x|^{-q}, w(x) = 1$ (Hardy inequality), and $v(x) = |x|^b$, $w(x) = |x|^a$ (Caffarelli-Kohn-Nirenberg [3]), and in all these cases we have a range of values of p and unique critical exponent $q = p^*$ for which the inequality holds.

In a recent work, Cabré and Ros-Oton (2013) [2] considered the case of identical monomial weights $v(x) = w(x) = x_1^{a_1} \cdot \ldots \cdot x_N^{a_N}$ establishing the validity of such inequality in when $a_i \geq 0$ for all $p \geq 1$. They show that if $D = N + a_1 + a_2 + \ldots + a_N$, then $q = p^* = \frac{Dp}{D-p}$.

In this talk we will show how to extend the result of Cabré and Ros-Oton to obtain a Hardy-Sobolev type inequality for different monomial weights, that is to consider weights of the form

$$v(x) = x_1^{b_1} \cdot \ldots \cdot x_N^{b_N} \quad \text{and} \quad w(x) = x_1^{a_1} \cdot \ldots \cdot x_N^{a_N},$$

obtaining conditions on p, q, a_i and b_i so that (1) holds.

References

[1] Castro, Hernán , *Hardy-Sobolev type inequalities for monomial weights*, preprint (2016).

[2] X. Cabré and X. Ros-Oton, *Sobolev and isoperimetric inequalities with monomial weights*, J. Differential Equations (2013).

[3] L. A. Caffarelli, R. V. Kohn, and L. Nirenberg, *First order interpolation inequalities with weights*, Compositio Math. (1984).