Harald Helfgott. Soficity, recurrence and short cycles of exponential maps.

Abstract. Let f be an exponentiation map mod p, or, more precisely, the map from $\{0, 1, \ldots, p-1\}$ to itself defined by $f(x) \equiv 2^x \mod p$. It is easy to show that f and $f \circ f$ have few fixed points. Showing that $f \circ f \circ f$ has $o(p)$ fixed points is harder, and was open; we show how to prove it. What about $f \circ f \circ f \circ f$? There, the problem is still open; we show its connection to *sofic groups*. More precisely: if the Higman group is sofic, then there is a map f that (a) behaves almost everywhere like an exponentiation map, and (b) satisfies $f(f(f(f(x)))) = x$ for almost all x. The proof rests in part on an elementary proof of a special case of the uniqueness of sofic representations of amenable groups. Joint work with K. Juschenko.
