Proof: (of Lemma 2.1)

By ~~[illegible]~~ Convexity $\Rightarrow D^2 f \geq 0$

In particular:

$$\Delta f \geq 0$$

Thus, by the maximum principle:

$$\sup_{x \in \Omega} f(x) \leq \sup_{x \in \partial\Omega} f(x) = 0$$

By definition $f \geq \hat{f}$, hence:

$$\inf_{x \in \Omega} f(x) \geq \inf_{x \in \Omega} \hat{f}(x) \geq -\|\hat{f}\|_{L^\infty}$$

Thus:

$$\|f\|_{L^\infty} = \mathrm{Max}\left(\sup_{x \in \bar{\Omega}} f(x), -\inf_{x \in \bar{\Omega}} f(x)\right) \leq \|\hat{f}\|_{L^\infty}$$

Lemma:

Let $\varphi_0, \varphi_1, \varphi_2 :]-\varepsilon, \varepsilon[\to \mathbb{R}$ be diff @ 0.

Spse that ~~$\forall t > 0$~~ $\varphi_0(0) = \varphi_1(0) = \varphi_2(0)$ & $\forall t > 0$:

$\varphi_0(t) \leq \varphi_1(t) \leq \varphi_2(t)$

Thus:

$$\varphi_0'(0) \leq \varphi_1'(0) \leq \varphi_2'(0)$$

Proof:

$\forall t > 0 \quad \dfrac{\varphi_0(t)}{t} \leq \dfrac{\varphi_1(t)}{t} \leq \dfrac{\varphi_2(t)}{t}$

$$\Rightarrow \frac{\varphi_0(t) - \varphi_0(0)}{t} \leq \frac{\varphi_1(t) - \varphi_1(0)}{t} \leq \frac{\varphi_2(t) - \varphi_2(0)}{t}$$

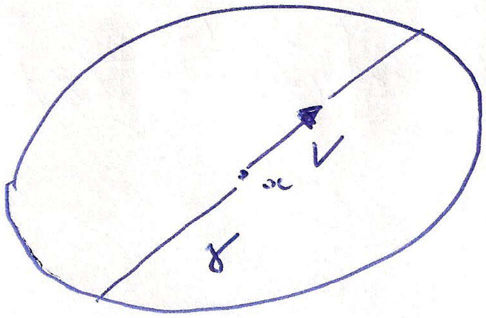
Taking limits yields:

$$\varphi_0'(0) \leq \varphi_1'(0) \leq \varphi_2'(0)$$

Proof of Lemma 6.3:

~~Let~~ $\cancel{x \in \Omega}$ Choose $x \in \Omega$. Choose $V \in \mathbb{R}^n \setminus \{0\}$.

Define $\gamma : \mathbb{R} \to \mathbb{R}^n$ by $\gamma(t) = x + tV$:



Since Ω is convex, $\cancel{\gamma^{-1}}$ $\exists\, a \leq 0 \leq b$ st

$\gamma^{-1}(\overline{\Omega}) = [a, b]$. By the chain rule:

$$(f \circ \gamma)''(t) = D^2 f(\gamma(t))(V, V) \geq 0 \quad \forall t$$

$$\Rightarrow (f \circ \gamma)' \uparrow$$

Thus, $\cancel{\text{st}\, f \circ \gamma}$:

$$\cancel{V(f\circ\gamma)} \quad (f \circ \gamma')(0) \leq (f \circ \gamma')(b)$$

Thus:

$$Df(x)(V) = Df(\gamma(0))(V)$$

$$\underset{\text{Chain Rule}}{=} (f \circ \gamma')(0)$$

$$\leq (f \circ \gamma')(b)$$

$$\cancel{\text{Chain Rule}} \; \leq (\hat{f} \circ \gamma')(b)$$

$$\text{– since } \hat{f} \leq f \leq 0$$