On the Saddlepoint Problem for Differentiable Multiobjective Optimization and Generalized Notions of Convexity

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Abstract

Saddle point theory has been of particular interest in research related to nonlinear programming, not only because it is an alternative way to establish optimality conditions, but also because it is intrinsically related with the existence of a solution for the problem under analysis. Over the years, many of the results of nonlinear programming were extended to multiobjective optimization problems, and saddle point theory has not been the exception. For these results to be really strong and meaningful, however, the functions involved in the problem must satisfy some convexity properties. In this contribution, the relationship between Pareto solutions, saddle points of a scalar Lagrangian function associated with the inequality constrained multiobjective optimization problem, and Fritz John critical points are presented under the well-known invexity condition whenever required. Besides, some generalized notions of convexity are revisited and the diagram originally devised by Ben Israel and Mond (1986) for scalar optimization, and improved by Giorgi (1990), has been extended to the multiobjective case.

Key words: Multiobjective optimization; Fritz John points; saddle points; efficient points; generalized convexity

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