EXISTENCE AND STABILITY OF THE TRAVELLING WAVE SOLUTIONS FOR A SEMI-LINEAR WAVE EQUATION.

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Resumo/Abstract:
This research focuses on the study of the semilinear wave equation

$$u_{tt} - \Delta u - u\,|u|^{p-2} = 0, \tag{1}$$

for $p(2, p^*]$, with p^* the critical Sobolev exponent and $n \geq 3$ The travelling wave solutions for (1) satisfy an equation of the form

$$-\sum_{i=1}^{n} \beta_i^2 v_{x_i x_i} + \sum_{j,i=1}^{n} c_i c_j v_{x_j x_i} - v\,|v|^{p-2} = 0. \tag{2}$$

The central part of this work consists in showing that there exist travelling wave solutions for the equation (1). This is done through a minimization problem, associated with a variational problem with restriction. Specifically, using the concentration-compactness principle of P.L. Lions [1], is established the existence of minimizers; by testing that the minimizing sequences - mod a subsequence- converge strongly in a functions space D. Where D is the completion of the space functions

$$H = \{v : \Re^n \to \Re | v \in C_0^\infty(R^n), v \in H_0^1(\Omega)\}, \tag{3}$$

with the norm

$$\| v \|^2 = \int_{\Re^n} |\nabla v(x)|^2 \, dx. \tag{4}$$

Also we prove, by using the concentration-compactness principle, that the set of minimizers is a stable set for an initial value problem associated; i.e., it is proved that the solutions for the equation (2), so also for the equation (1), are stable respecting to a set of minimizing functions.

References

[1] P. L. Lions, *The concentration-compactness principle in the calculus of variations. The limit case, part 1* , Rev. Mat. Iberoamericana, 1985.