Neumann Boundary Controllability of the Gear–Grimshaw System With Critical Size Restrictions on the Spacial Domain

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Resumo/Abstract:

In this paper we study the boundary controllability of the Gear-Grimshaw system posed on a finite domain $(0, L)$, with Neumann boundary conditions:

$$\begin{cases} u_t + uu_x + u_{xxx} + av_{xxx} + a_1 vv_x + a_2(uv)_x = 0, \\ cv_t + rv_x + vv_x + abu_{xxx} + v_{xxx} + a_2 buu_x + a_1 b(uv)_x = 0, \\ \\ u_{xx}(0,t) = h_0(t),\ u_x(L,t) = h_1(t),\ u_{xx}(L,t) = h_2(t), \\ v_{xx}(0,t) = g_0(t),\ v_x(L,t) = g_1(t),\ v_{xx}(L,t) = g_2(t), \\ \\ u(x,0) = u^0(x), \quad v(x,0) = v^0(x). \end{cases} \tag{1}$$

where $(x, t) \in (0, L) \times (0, T)$ We first prove that the corresponding linearized system around the origin is exactly controllable in $(L^2(0, L))^2$ when $h_2(t) = g_2(t) = 0$. In this case, the exact controllability property is derived for any $L > 0$ with control functions $h_0, g_0 \in H^{-\frac{1}{3}}(0, T)$ and $h_1, g_1 \in L^2(0, T)$. If we change the position of the controls and consider $h_0(t) = h_2(t) = 0$ (resp. $g_0(t) = g_2(t) = 0$) we obtain the result with control functions $g_0, g_2 \in H^{-\frac{1}{3}}(0, T)$ and $h_1, g_1 \in L^2(0, T)$ if and only if the length L of the spatial domain $(0, L)$ belongs to a countable set. In all cases the regularity of the controls are sharp in time. If only one control act in the boundary condition, $h_0(t) = g_0(t) = h_2(t) = g_2(t) = 0$

and $g_1(t) = 0$ (resp. $h_1(t) = 0$), the linearized system is proved to be exactly controllable for small values of the length L and large time of control T. Finally, the nonlinear system is shown to be locally exactly controllable *via* the contraction mapping principle, if the associated linearized systems are exactly controllable.

References

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