The problem of best fitting a curve to a set of data points on a Riemannian manifold will be addressed. We start with geodesic fitting, which is a direct generalization of linear regression to the manifold setting, and then present a variational approach to tackle generalized least squares problems on Riemannian manifolds. These problems, which are motivated by many real applications, are also theoretically very challenging. The Grassmann manifold will serve as a benchmark example to illustrate the proposed methods and highlight the geometry of the problems and the complexity of the solutions.