UNIQUENESS OF QUASI-EINSTEIN METRICS ON 3-DIMENSIONAL HOMOGENEOUS RIEMANNIAN MANIFOLD

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The study of 3-dimensional homogeneous Riemannian manifolds is done, in general, according to the dimension of its isometry group $Iso(M^3, g)$, which can be 3, 4 or 6. One of the motivation to study m-quasi-Einstein metrics on a Riemannian manifold $(M^n,\ g)$ is its closed relation with warped product Einstein metrics, see e.g. [2]. For instance, when m is a positive integer, m-quasi-Einstein metrics correspond to exactly those n-dimensional manifolds which are the base of an $(n+m)$-dimensional Einstein warped product. Following this trend we present here a complete description of m-quasi-Einstein metrics, that are non gradient, when this manifold compact or not compact provided $dim\, Iso(M^3, g) = 4$. In addition, we shall show the absence of such structure on Sol^3, which corresponds to $dim\, Iso(M^3, g) = 3$. When $dim\, Iso(M^3, g) = 6$ it is well known that M^3 is a space form. In this case, its canonical structure gives a trivial example. In particular, we shall prove that Berger's spheres carry naturally a non trivial structure of quasi-Einstein metrics. Since they have constant scalar curvature, their associated vector fields can not be gradient. In particular, we can not extend Perelman's result to compact quasi Einstein metrics. Moreover, these examples show that Theorem 4.6 of [3] can not be extended for a non gradient vector field.

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