A Free Boundary Problem Related to Thermal Insulation

Abstract

We consider a variational problem for domains coming from the task of finding an optimal thermal insulator. Let $\Omega \subseteq \mathbb{R}^n$ be a (nice) fixed set, and minimize

$$F(A, u) = \int_A |\nabla u|^2 d\mathcal{L}^n + h \int_{\partial A} u^2 d\mathcal{H}^{n-1} + C_0 \mathcal{L}^n(A \setminus \Omega)$$

over all sets A containing Ω and having smooth boundary, and all smooth functions $u \in C^1(A)$ with $u \equiv 1$ on Ω. Here h and C_0 are fixed, positive parameters. This may be thought of as a variational free boundary problem, with the unusual characteristic that along the boundary of the minimal set, ∂A, the harmonic function u satisfies a Robin condition, not the typical Dirichlet condition. We will briefly explain how this problem arises, discuss how an appropriately relaxed version of our functional admits minimizers, and then describe the regularity properties of these minimizers. This is based on joint work with Luis Caffarelli.