A class of adding machines and dynamics in $\mathbb{R}^2$

Danilo Antonio Caprio
UNESP-IBILCE, Departamento de Matemática
Rua Cristovão Colombo, 2265, CEP 15054-000, São José de Rio Preto-SP, Brasil
Supported by FAPESP grant 2015/26161–6
danilocaprio@sjrp.unesp.br

ABSTRACT

In this work I will define the stochastic adding machine associated to the Fibonacci base $(F_n)_{n\geq 0}$ (where $F_0 = 1$, $F_1 = 2$ and $F_n = F_{n-1} + F_{n-2}$, for all $n \geq 2$) and to the probabilities sequence $(p_i)_{i\geq 1}$. I will consider the transition operator S and I will prove that the Markov chain is transient if and only if $\prod_{i=1}^{\infty} p_i > 0$. Otherwise, if $\sum_{i=1}^{+\infty} p_i = +\infty$, then the Markov chains is null recurrent and if $\sum_{i=2}^{+\infty} p_i F_{2(i-1)} < +\infty$, then the Markov chain is recurrent positive.

I will compute the point spectrum and prove that it is connected to the fibered Julia sets for a class of endomorphisms in $\mathbb{C}^2$. Precisely $\sigma_{pt}(S) \subset E \subset \sigma(S)$ where $E = \{z \in \mathbb{C} : (g_n \circ \ldots \circ g_0(z,z))_{n\geq 1}$ is bounded$\}$ and $g_n : \mathbb{C}^2 \longrightarrow \mathbb{C}^2$ are maps defined by $g_0(x,y) = \left(\frac{x-(1-p_1)}{p_1}, \frac{y-(1-p_1)}{p_1}\right)$ and $g_n(x,y) = \left(\frac{1}{r_n}xy - \left(\frac{1}{r_n} - 1\right), x\right)$ for all $n \geq 1$, where $r_n = p_{\left[\frac{n+1}{2}\right]+1}$. Moreover, if $\liminf_{i\to+\infty} p_i > 0$ then E is compact and $\mathbb{C} \setminus E$ is connected.

Finally, I will prove that $\sigma_{pt}(S) \cap \mathbb{R} = E \cap \mathbb{R}$.

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