Financial Instability Contagion: modeling and data calibration

[Preliminary Report]

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Abstract

This is a continuation of our ongoing research on financial market instability. Previously we established an early warning system called the *market instability indicator* and a quantitative definition of financial instability contagion. In this paper we use macroeconomic data of the United States and selected Eurozone countries to test the practicality of the indicator and search empirical evidence of financial instability contagion in recent financial crises, the U.S. subprime crisis and the Eurozone sovereign credit crisis. We also discuss the limits of available data and modifications that are necessary to implement theoretical models in real life.

Keywords multi-agent models, financial instability, systemic risk, contagion

1 Introduction

Since the breakout of the U.S. subprime-originated financial crisis in 2007, the terms 'systemic risk' and 'contagion' have been mentioned regularly in economic and financial circles, and with the sovereign credit crisis in the Eurozone, they almost became a household vocabulary. But what exact is contagion and how is it modeled when studying financial crises? Is every financial crisis accompanied by contagion, and if so, is there any early warning system that predicts it?

Choi and Douady [5, 6] used flow of funds analysis of multi-agent model for a single economy to build a dynamical system of wealth and established an early warning system called the *market instability indicator*. Choi and Castellacci [3] extended this result to a system of multiple economies and gave a quantitative definition of financial instability contagion.[1] In this article we use real data to verify the validity of the market instability indicator and contagion mechanism. We apply the theory to recent financial crises, the U.S. subprime crisis and the Eurozone sovereign credit crisis, and as such, calibrate data of the United States and selected Eurozone countries. Transition from theory to reality is not smooth most of the time, unfortunately. Quite often required data are not frequent enough if not available at all, hence it is necessary that the models be modified. Whenever such modifications are made, we keep a list of lacking data components and possible mismatch between our data calibration and historical events. In doing so, we will also compare the data format and availability of the countries under investigation, address their shortcomings, and make suggestions for future improvement.

[**Redo this part.**] The rest of the paper is organized as follows. In Section 2 we summarize the previous theoretical results that form the foundation of data calibration. In Section 3, we first discuss the available sources of data and their limits, then we show a selection of calculated market instability indicators for the United States and their implication in financial market instability and contagion. **[More is to be added, including the result for the Eurozone.**]

2 Theoretical Results to Date

2.1 Economic Agents and Flow of Funds

We divide an economy into n large aggregates, commonly called sectors, that we assume to act as economic agents. At each time t we observe the wealth vector $w(t) = (w_1(t), ..., w_n(t)) \in \mathbb{R}^n$ where w_i represents the wealth of agent i. The selection of agents can vary depending on the economy and time under consideration. Castellacci and Choi [3] extended this single-economy model to a global economy that consists of unspecified s subeconomies. In [4] they used twenty agents to model the financial instability contagion in a mini Eurozone of four countries, Greece, Spain, France, and Germany.

The composition of each agent can change too. In [5] the government category was left out since it was considered as a super-systemic moderator of last resort, yet in [6], which models a complete domestic economy, government is included as an agent. Furthermore the central bank is included in the government category since the two entities tend to act together in the time of a financial distress. On the other hand, in [3] and [4] the central bank is treated as a super-systemic entity, the role played by government in [5], and left out. Note that the central bank in those articles is a super-national monetary authority like the European Central Bank that controls monetary policy of the global economy and not the central banks of individual subeconomies.

Once we select the agents, we define the wealth of each agent as the sum of equity

[1] From now on we simply say 'contagion' to mean contagion of financial instability.

and debt[2], and also the sum of liquidities (cash and cash-equivalent - produces no income) and invested assets (bank deposit, financial securities, real estate etc. - produces income such as interest, capital gain). Hence the wealth $w_i(t)$ of agent i at t is

$$w_i(t) = E_i(t) + D_i(t) = L_i(t) + K_i(t) \tag{1}$$

where $E_i(t)$, $D_i(t)$, $L_i(t)$, and $K_i(t)$ are the equity, debt, liquidities, and invested assets, respectively, of i at t.

The agents interact through cash flows between them, which roughly classified into two groups, *at-will (variable)* and *fixed*, depending on the obligation type of the payer. Examples of variable flow of funds are equity and debt investments, consumption, dividend payment etc. Fixed flow of funds include wages, coupons, mortgage payments etc. A third type is *contingent* such as monetary authorities' emergency liquidity injection. Also there are *international* flow of funds when there are more than one countries are involved. Detailed explanation on flow of funds and feedback loops can be found in [3], [4], [5] and [6].

We denote by $F_{ij}(t) > 0$ the fund transferred from agent j to i at time t to changes the wealth w_i of i. There are two ways of obtaining F_{ij}, as an optimal solution of a nonlinear programming problem (NLP) [**?**] [**?**] and by observing historical time series such as [10]. Choi and Douady [5] [6] followed the former approach and assumed that F_{ij} optimizes some utility of each agent subject to constraints on liquidity and borrowing capacity. Choi and Castellacci [3] followed the latter assuming that the observed time series are the result of the same NLP. In other words the steps to solve NLP were skipped and only the result was taken. In this article we follow the approach of [3] by obtaining the flow of funds data from historical time series, yet we keep the same mathematical assumptions in [5] and [6] when applying theories of dynamical systems to analyze financial market instability.

The change of the liquidities level of i can be written 'cash in' less 'cash out,'

$$\Delta L_i = \sum_{j \neq i}^{n} F_{ij}(t) - \sum_{k \neq i}^{n} F_{ki}(t) - \tilde{\Delta} K_i(t+1) \tag{2}$$

where $\tilde{\Delta} K_i(t)$ is new investment less liquidation, i.e. net conversion from L_i to K_i. The invested asset next time is

$$K_i(t+1) = (1 + \gamma_i(t))\, K_i(t) + \tilde{\Delta} K_i(t+1) \tag{3}$$

where $\gamma_i(t)$ is the aggregate return on $K_i(t)$. Here aggregate return means the average return on all combined investments.

[2]In this sense 'asset' is a more familiar term for our 'wealth.'

We observe $L_i(t)$, $K_i(t)$, and $D_i(t)$ from data and assume they evolve as follows:[3]

$$D_i(t+1) = (1 + r_i(t))D_i(t) + \tilde{\Delta} D_i(t+1) \tag{4}$$
$$K_i(t+1) = (1 + \gamma_i(t))K_i(t) + \tilde{\Delta} K_i(t+1) \tag{5}$$
$$L_i(t+1) = L_i(t) + \Delta L_i(t+1) \tag{6}$$

With Equations (1), (2), and (6),

$$w_i(t+1) = w_i(t) + \gamma_i(t)K_i(t) + \sum_{j\neq i}^{n} F_{ij}(t) - \sum_{k\neq i}^{n} F_{ki}(t). \tag{7}$$

The internal return $\gamma_i(t)K_i(t)$ on the invested asset $K_i(t)$ can be interpreted as a result of 'self-growth,' hence we replace it by

$$F_{ii}(t) = \gamma_i(t)K_i(t). \tag{8}$$

Hence Equation (8) becomes

$$w_i(t+1) = w_i(t) + F_{ii}(t) + \sum_{j\neq i}^{n} F_{ij}(t) - \sum_{k\neq i}^{n} F_{ki}(t) \tag{9}$$

and finally

$$w_i(t+1) = w_i(t) + \sum_{j=1}^{n} F_{ij}(t) - \sum_{k\neq i}^{n} F_{ki}(t) \tag{10}$$

We denote by $w(t)$ the wealth of the economy,

$$w(t) = (w_1(t), w_2(t), \ldots, w_n(t)). \tag{11}$$

2.2 Elasticity and Market Instability Indicator

We now track the evolution of $w(t)$. First we collect w_i and F_{ij} in constant dollars.[4] Then define a discrete wealth dynamical system $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ by

$$f(w_i(t)) = w_i(t+1). \tag{12}$$

This dynamical system is built retrospectively, thus deterministic.[5] The dynamical system $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$ has a Jacobian matrix $Df(t) = B(t) = \left(b_{ij}(t)\right)_{1\leq i,j\leq n}$

$$b_{ij} = \frac{\partial w_i(t+1)}{\partial w_j(t)} \text{ [6]} \tag{13}$$

[3]The notation $\tilde{\Delta}$ is different from the ordinary time-increments, e.g., $\Delta D_i(t) = D_i(t+1) - D_i(t)$. Note that [3], [4] used $\tilde{\Delta}(t+1)$ and [5], [6] used $\Delta D_i(t+1)$ for our $\Delta \tilde{D}_i(t)$.

[4]This can be achieved by the GDP deflator or the money market account evaluated from a base year.

[5]A different approach was used in [5], [6] in which case the resulting dynamical system is a predictable process.

[6]Regardless of the differentiability of f, we use the partial derivative notation, which may, depending on f, actually mean the difference quotient $\frac{\Delta w_i(t+1)}{\Delta w_j(t)}$. See [3] for conditions on the differentiability of f.

We now define the *elasticity coefficient* that measures the impact of the wealth change of an agent to its outgoing cash flow. All variables being measured at time t, let agent j experience a change dw_j in its wealth and let a_{ij} be the elasticity coefficient of its payment to another agent i, then the cash flow from j to i is changed by:

$$dF_{ij} = a_{ij} dw_j \tag{14}$$

or equivalently,

$$a_{ij} = \frac{\partial F_{ij}}{\partial w_j}. \tag{15}$$

In case $i = j$, we observe 'self-elasticity' a_{ii} of i that results from the auto-correlation of its internal return on invested assets K_i. It represents the variation of its wealth at time $t+1$ upon a change of it at time t.

$$a_{ii}(t) = \frac{\partial F_{ii}(t)}{\partial w_i(t)} = \frac{\partial\left(\gamma_i(t) K_i(t)\right)}{\partial w_i(t)} \tag{16}$$

In any case, we assume that the change of wealth dw_j purely affects the cash flow $F_{ij}(t)$.

$$\frac{\partial F_{ij}(t)}{\partial w_k(t)} = 0 \text{ if } j \neq k \tag{17}$$

In other words, a change in the wealth of one agent doesn't have an immediate impact on flows of funds between two other agents. Hence we have $a_{ij} dw_j = dF_{ij}$ for all $1 \leq i, j \leq n$, including $j = i$.

The elasticities a_{ij} and the Jacobian b_{ij} are closely related. Choi and Douady [5] showed that they are different only on the diagonal:

$$b_{ii} = 1 + a_{ii} - \sum_{k \neq i}^{n} a_{ki} \quad \text{and} \tag{18}$$

$$b_{ij} = a_{ij} \qquad \text{for } i \neq j \tag{19}$$

Based on well-known theories of dynamical systems, Choi and Douady [5] defined a *market instability indicator* $I(t)$ as the spectral radius of the Jacobian matrix $B(w(t))$, i.e. the modulus of its largest eigenvalue.

$$I(t) = \rho\left(B(w(t))\right). \tag{20}$$

The instability indicator is a time dependent, local indicator that can be estimated empirically, using the estimated elasticities. By monitoring the instability indicator, we can track whether the market is moving toward an equilibrium and tends to absorb perturbations, or away from one and amplifies exogenous shocks.

2.3 Dynamical System of Wealth for Multiple Economies

The result for single economy model in Sections 2.1 and 2.2 can be extended to a multi-economy system as detailed in [3]. When a global economy consists of a collection of s subeconomies such that the subeconomy k is divided into n_k agents, we build the *global wealth vector* $w(t) = (w_1(t), ..., w_n(t)) \in \mathbb{R}^n$, where $n = \sum_{k=1}^{s} n_k$, as a canonical embedding of the respective wealth vector of each economy, $w^k(t) = (w^k_1(t), w^k_2(t), \ldots, w^k_{n_k}(t))$, where $w^k_j(t)$ is the wealth of agent j of economy k at time t. Therefore $w_i(t) = w^k_j(t)$ if

$$i = N(k) + j, \qquad N(k) = \sum_{l=1}^{k-1} n_l \tag{21}$$

As a result, $w_i(t)$ for each i inherits the properties of $w^k_j(t)$ and the definitions of wealth dynamical system $f : \mathbb{R}^n \longrightarrow \mathbb{R}^n$, Jacobian matrix and elasticities can be extended to the global economy. To show how the global system and subeconomies are related, we extend the indexing in Equation (21). The cash flow $F_{ij}(t)$ is a cash flow from agent j to i at time t. When two agents i and j belong to the same subeconomy, say k, and we need to focus on that particular one, we specify it by an upper index:

$$F_{N(k)+i,N(k)+j}(t) = F^k_{ij}(t). \tag{22}$$

Likewise, we write the Jacobian matrix $B^{(k)}(t) = \left(b^k_{ij}(t)\right)$ and the elasticity matrix $A^{(k)}(t) = \left(a^k_{ij}(t)\right)$ of subeconomy k, respectively. To track the interaction between agents from different subeconomies, we use upper index for the subeconomies and lower index for the agents. Thus we denote the cash flows from agent j of economy l to agent i of subeconomy k at time t by $F^{kl}_{ij}(t)$, which is in terms of the global system can be written as

$$F^{kl}_{ij}(t) = F_{N(k)+i,N(l)+j}(t) \tag{23}$$

Therefore

$$a^{kl}_{ij}(t) = \frac{\partial F^{kl}_{ij}(t)}{\partial w^l_j(t)} \tag{24}$$

where $w^l_j(t)$ is the wealth of agent j in subeconomy l.

Both the local elasticity matrices $A^{(k)}(t)$ and local Jacobian matrices $B^{(k)}(t)$ can be canonically embedded into the global elasticity matrix $A(t)$ and global Jacobian matrix

$B(t)$. respectively. By Equation 18

$$B(t) = \left(\begin{array}{c:c:c:c} B^{(1)}(t) & A^{(12)}(t) & \dots & A^{(1s)}(t) \\ \hdashline A^{(21)}(t) & B^{(2)}(t) & & \\ \hdashline \vdots & & \ddots & \\ \hdashline A^{(s1)}(t) & \dots & & B^{(s)}(t) \end{array}\right). \tag{25}$$

Subeconomies k and l interact via international flow of funds such as international investment and trades (see [3] for details), therefore $F^{kl}_{ij}(t) \neq 0$ for some i and j. Consequently some off-diagonal blocks of $B(t)$ are not zeros, and this is an important condition in defining the contagion of financial instability.

2.4 Quantitative Definition and Mechanism of Contagion

The following definition of financial instability contagion, based on the market instability indicator $I(t)$, is due [3]. The idea is that initially both the subsystems and global system are stable, then at least one of the subsystems enters a state of financial instability, which eventually propagates to the entire system by *causality*. Mathematically speaking:

Definition 1 (Contagion)**.** *We say that* contagion *in a global economic system occurs if given two time instants* t_0, t_1 *with* $0 < t_0 < t_1$ *one has*

(i) At time $t < t_0$, $\max_k \rho(B^{(k)}(t)) < 1$ *and* $\rho(B(t)) < 1$

(ii) At time $t \in (t_0, t_1)$, $\max_k \rho(B^{(k)}(t)) > 1$ *and* $\rho(B(t)) < 1$

(iii) At time $t > t_1$ $B(t) \neq \oplus_{k=1}^{s} B^{(k)}(t)$ *and* $\rho(B(t)) > 1$.

Above definition can be used for subeconomies belonging to different currency zones by converting the flow of funds into a single major currency such as the US dollar or the euro. We can also apply above definition to contagions within a single-economy system by replacing subsystems by partitions of the economy. Note that contagion as defined is different from "domino effect," which describes a phenomenon in which each subeconomy takes turns to become unstable, i.e. $\rho(B^{(i)}(t)) > 1, \rho(B^{(j)}(t)) < 1$ and $\rho(B^{(i)}(s)) < 1, \rho(B^{(j)}(s)) > 1$ for $i \neq j$ and $t \neq s$ while $\rho(B) < 1$ for all times.

The same article [3] explains the mechanism of contagion using a lower bound of the market instability indicator $\rho(t)$. For an n-agent economic system,

$$\frac{\sum_{i=1}^{n} |b_{ii}(t)|}{n} = \left| 1 + \frac{1}{n} \sum_{i=1}^{n} a_{ii}(t) - \frac{1}{n} \sum_{i \neq k} a_{ik}(t) \right| \leq \rho(t). \tag{26}$$

In [3] are explained in detail three common types of contagion: within an economy, cross-boarder contagion by default, and cross-boarder contagion by fear factor. They are all based on the phenomenon that when an agent, say i, is highly leveraged, a sudden negative shock on its wealth w_i[7], can make some of its outgoing cash obligation, especially the ones related to debt payment, increase while cash inflows decrease due to loss of confidence and fleeing investors. Mathematically speaking, some F_{ij} decreases and F_{ik} increases in Equation (10), and this being happening when $dw_i < 0$, the elasticity a_{ik} is negative. As the wealth w_i continues decreasing, the magnitude $|a_{ki}|$ increases with acceleration. As a result $|b_{ii}| = |1 + a_{ii} - \sum_{k \neq i} a_{ki}|$ will go up as well. If agent i belongs to a partition l of an economy (or a subeconomy of a multi-economy system), then increasing $|b_{ii}|$ could push the indicator $\rho(B^{(l)}(t))$ of the subeconomy l above 1, which could, depending on the structure of the feedback loop of the global economy, make the negative wealth shock dw_i propagate outside of subeconomy l. Using $\frac{\sum_{i=1}^{n} |b_{ii}(t)|}{n}$ as a lower bound of $\rho(B(t))$ is useful in estimating the market instability indicator with real data because the wealth $w_i(t)$ of each agent is easier to obtain (e.g. Table B of [10]) than the entire set $F_{ij}(t)$ of flow of funds, and by definition,

$$b_{ii}(t) = \frac{\partial w_i(t+1)}{\partial w_i(t)}. \tag{27}$$

Hence by obtaining b_{ii} from w_i and using Equation (26), we can get one-sided information of $\rho(B(t))$ without calculating the indicator itself. In the following section, we use historical time series of the United States **[for now. More countries will be added later]** to verify the validity of this contagion mechanism and prove that there was indeed financial instability contagion, in the sense of [3], in the 2007-09 U.S. subprime crisis.

3 Data Calibration

3.1 Data Collection and Adaptation

When using real data one often has to rely on existing classification of agents, since it is almost impossible to reorganize available data that already may not be sufficient. In case of the United States, the main source of data is the *Integrated Macroeconomic Accounts for the United States (IMA)* jointly published by the Federal Reserve Board (Fed) and the Bureau of Economic Analysis (BEA) [10, 2]. The two agencies used to have separate set of data that were not always compatible, yet developed the IMA several years ago to integrate the data sets to fill information gaps and enhance international compatibility of the US national accounts. As a result it is on par with the international accounting standard, System of National Accounts 2008 (SNA 2008) that most countries follow. One major difference between the IMA and SNA 2008 is the selection of agents. In IMA, there are total six economic agents, Household and Nonprofit Organizations (HNO)[8], Nonfinancial Noncorporate Business (NFNC),

[7] In reality such events have been warned about at least. In case of the 2007-09+ US subprime crisis for example, there were plenty of ominous events [1] before the demise of Lehman Brothers and there were even people who bet on the forthcoming crisis.

[8] Nonprofit organization here includes hedge funds, private equity funds, and private trusts.

Nonfinancial Corporate Business (NFC), Financial Business (FB)[9], State and Local Government (SLG), Federal Government (FG) and Rest of the World (ROW). On the other hand, SNA 2008 uses five agents, Households including NPISH[10] (S.14+S.15), General government (S.13), Financial corporations (S.12), Non-financial corporations (S.11), and the Rest of the World (S.2). The five Eurozone countries whose data we have looked follow SNA 2008, and their major data source are the central banks and national statistical institutions:

- France: Banque de France, Institut National de la Statistique et des Études Économiques (INSEE)
- Germany: Deutsche Bundesbank, Destatis-Statistisches Bundesamt
- Greece: Bank of Greece, Hellenic Statistical Authority (EL.STAT.)
- Ireland: Bank of Ireland, Central Statistics Office
- Spain: Banco de Espanã, Instituto Nacional de Estadística

For all five countries, quarterly financial accounts are available from the respective central banks. For non-financial accounts, however, little is available. For Spain and Ireland, only quarterly capital accounts and no revaluation accounts are publicly available. This means that the book value of assets are used for the end-of-period balance sheet, while our model requires the market value to accurately estimate the elasticities and market instability indicator. For Germany sectorial balance sheet is available only annually, which is not frequent enough, and for France and Greece, nothing has been found so far. This lack of data is the major shortcoming of the Eurozone countries under investigation compared with the United States. Nevertheless we have used what data available at our best.

3.2 U.S. Subprime Crisis 2007-2009

The U.S. subprime crisis seems to have all the components that a financial crisis could have: an asset bubble and subsequent burst of it, a financial crisis followed by a period depression, an intervention of the government with unforseen monetary policy that still continues to date. We track the evolution of the crisis by considering the U.S. economy as a global one that consists of six subeconomies, i,e, the domestic agents (HNO, NFNC, NFC, FB, SLG, and FG) and calculating their respective market instability indicator. In mathematical terms, it is $|b_{ii}|$ of the Jacobian matrix of the wealth dynamical system f. Because $b_{ii}(t) = \dfrac{\partial w_i(t+1)}{\partial w_i(t)}$ be definition, we will take only necessary components from available data as opposed to building complete Df.

[9]Monetary authority that includes the Federal Reserve Bank and the Treasury Monetary Account but excludes the Federal Reserve Board Account belongs to the financial business category.

[10]Nonprofit Institutions Serving Households

For all six domestic agents, we use the IMA quarterly data tables S.3.q - S.8.q. [2] For the rest of the article, we use prime '$\prime$' to denote derivative with respect to time. Recall Equation (9),

$$w_i(t+1) = w_i(t) + F_{ii}(t) + \sum_{j\neq i}^{n} F_{ij}(t) - \sum_{k\neq i}^{n} F_{ki}(t).$$

By differentiating all terms with respect to time, we get

$$\begin{aligned} w_i'(t+1) &= \sum_j \frac{\partial w_i(t+1)}{\partial w_j(t)} \frac{dw_j(t)}{dt} \\ &= \sum b_{ij}(t)\, w_j'(t) = b_{ii}(t)\, w_i'(t) + \sum_{j\neq i} b_{ij}(t)\, w_j'(t) \\ &= \sum b_{ij}(t)\, w_j'(t) = b_{ii}(t)\, w_i'(t) + \sum_{j\neq i} a_{ij}(t)\, w_j'(t) \end{aligned}$$

by Equation (19). By Equation (17), $a_{ij} = \dfrac{F'_{ij}(t)}{w'_j(t)}$, hence

$$w_i'(t+1) = b_{ii}(t)\, w_i'(t) + \sum_{j\neq i} F'_{ij}(t) \tag{28}$$

or equivalently

$$\frac{w_i'(t+1)}{w_i'(t)} = b_{ii}(t) + \sum_{j\neq i} \frac{F'_{ij}(t)}{w_i'(t)}. \tag{29}$$

Assign indices 1 to 6 to HNO, NFNC, NFC, FB, SLG, and FG, in that order. For all agents, the cash inflows F_{ij} are available from IMA data except for loans: in the balance sheet accounts of Tables S.i.q ($3 \leq i \leq 8$) are provided only 'outstanding' liabilities while Equation (28) requires 'new loans made' excluding debt payment. Therefore we rewrite Equation (29) as follows. We assume all loans are made by banks or other financial institutions that act like banks, hence for agent other than FB, i.e. for $i \neq 4$,

$$\begin{aligned} \frac{w_i'(t+1)}{w_i'(t)} &= b_{ii}(t) + \sum_{j\neq i} \frac{F'_{ij}(t)}{w_i'(t)} \\ &= b_{ii}(t) + \sum_{j\neq i,4} \frac{F'_{ij}(t)}{w_i'(t)} + \frac{F'_{i4}(t) - F'_{4i}(t)}{w_i'(t)} + \frac{F'_{4i}(t)}{w_i'(t)} \\ &= b_{ii}(t) + \sum_{j\neq i,4} \frac{F'_{ij}(t)}{w_i'(t)} + \frac{(\text{net new loans})'(t)}{w_i'(t)} + a_{4i}(t), \end{aligned}$$

therefore

$$b_{ii}(t) = \frac{w_i'(t+1)}{w_i'(t)} - \sum_{j\neq i,4} \frac{F'_{ij}(t)}{w_i'(t)} - \frac{(\text{net new loans})'(t)}{w_i'(t)} - a_{4i}(t). \tag{30}$$

For FB, cash inflows other than operating surplus, i.e. the investment of other agents in FB, act like loans to FB, so Equation (30) is modified as follows,

$$b_{44}(t) = \frac{w_4'(t+1)}{w_4'(t)} - \frac{(\text{operating surplus})'(t)}{w_4'(t)} - \frac{(\text{net new loans})'(t)}{w_4'(t)} - \sum_{j \neq 4} a_{j4}(t). \quad (31)$$

With real data, we use difference quotient of variables over a single time period for the time derivative in Equations (30) and(30),. As for the elasticities a_{4i} and $\sum_{j\neq 4} a_{j4}$, we can only provide suggestions which, after many trial and errors, seem to fit best both the definition of elasticity coefficient - impact of wealth change to outgoing cash obligation - and historical events. In other words, we try to find what can serve as elasticity coefficients in real life.[11] In Equations (30) and (30) all the components but a_{4i} and $\sum_{j\neq 4} a_{j4}$ are already available from IMA data, hence how we estimate a_{4i} and $\sum_{j\neq 4} a_{j4}$ changes the level of financial instability of each agent.

One suggestion is to use the 'bad loans' data from the Federal Deposit Insurance Corporation (FDIC) [9]. Here 'bad loans' mean loans 30 or more days overdue or in nonaccrual status: loans are not paid on time implies that there is an impact of changes in wealth on outgoing cash obligations, and the impact can be measured by the 'default'[12] amount to wealth change ratio, i.e. $a_{4i} = \frac{\text{'default amount'}}{\Delta w_i}$. This approach can be used for HNO and combined nonfinancial business (NFB - assigned index 8[13]), since we cannot get separate information on bad loans for NFNC and NFC from the FDIC loan performance data.

Another suggestion is to use capital requirement in the form of equity-to-asset (E/A) ratio. This is especially useful for FB because FDIC data do not give realistic information on the financial health of FB: banks have sources of emergency funding such as the Federal Reserve's discount window; small banks may go under, yet big ones always seem too big to fail! Therefore, aggregately, FB rarely defaults on their loan payment. After all a substantial component of the Third Basel Accord (Basel III) is to keep a certain level of equity. In our calculation, excessive debt level beyond the preset capital requirement is considered 'default.' Three values of E/A ratio are selected for private sector agents. For HNO, we have found that the 'bad loan' approach gives almost identical result as E/A ratio of 0.8 and 0.82 (Figure 1), so the two methods could be used interchangeably indeed.

Remark 3.1. *In Equation (9)* $w_i(t+1)$ *and* $w_i(t)$ *correspond to the asset level of agent i at time t, and* $F_{ij}(t)$ *the flow incurred between the two times t and* $t+1$*. For IMA, the*

[11] After all, the goal of this paper is to verify with real data the validity of our theoretical models and not to calculate the exact values of elasticity coefficients. The latter is beyond the scope of this article and another ongoing project [7].

[12] This is not a legal sense of default, e.g. Chapter 11, but means 'failed to pay.' See [5, 6] for details on using the term.

[13] Index 7 is reserved for Rest of the World (ROW).

level L and flow F are related such that

$$L(t+1) = L(t) + F(t+1), \tag{32}$$

hence there is a shift of time for all flows F_{ij}, and the definition of b_{ii} and a_{ij} should be adjusted accordingly.

The following figures show the movement of the market instability indicator $|b_{ii}|$ for a decade or so around the 2007-09 subprime crisis with the elasticity coefficients a_{4i} (for HNO, NFNC, NFC, and NFB) and $\sum_{j\neq 4} a_{j4}$ (for FB) calculated as described above. Those for FG and SLG are assumed to be zero, because unlike the Eurozone countries, the federal government of the United States is not subject to default. Although state and local governments can default and some actually did, we assume, at least for now, the elasticity coefficient a_{46} is zero. Despite the rudimentary calculation, the behavior of the indicators correctly shows events that caused financial distress, such as the 911 terrorist attack and the meltdown of the housing market in the early 2007. Note that a spike of the indicator alone does not imply financial instability contagion. The elasticity coefficients at that time should also be examined, for they determine the causal nature of contagion (condition (iii) of Definition 1).

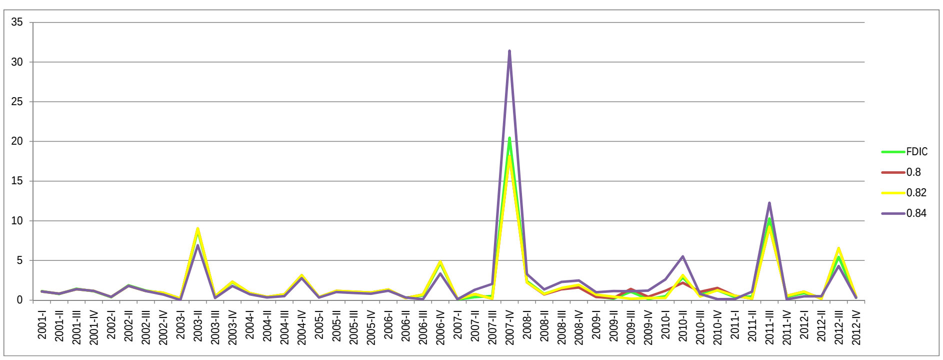

Figure 1: The market instability indicator $|b_{11}|$ for HNO with a_{41} calculated from 'bad loans' from FDIC data, and E/A ratio of 0.8, 0.82, and 0.84. We can see the indicators calculated from 'bad loans' and E/A ratio 0.8 and 0.82 are almost identical.

[**More writings on the elasticities as evidence of contagion are to be filled out.**]

3.3 Selected Eurozone Countries

[**More to follow.**]

4 Conclusion

To be filled out.

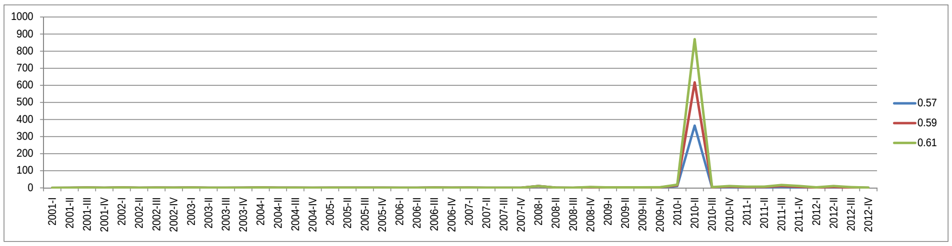

Figure 2: The market instability indicator $|b_{22}|$ for NFNC with a_{42} calculated from E/A ratio 0.57, 0.59, and 0.61.

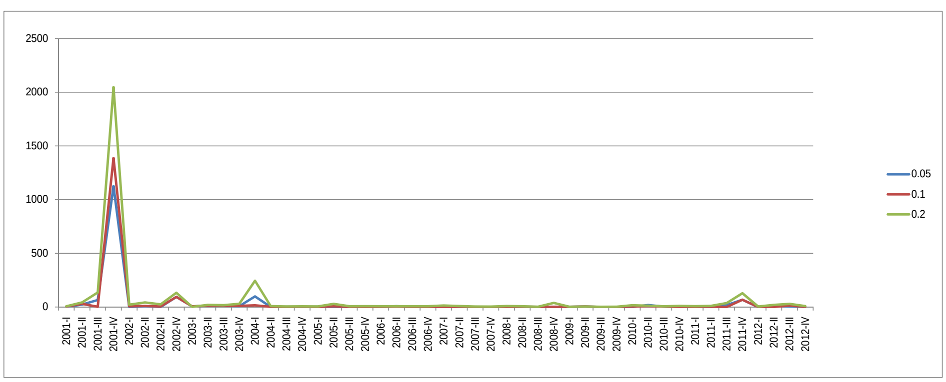

Figure 3: The market instability indicator $|b_{33}|$ for NFC with a_{43} calculated from E/A ratio 0.05, 0.1, and 0.2.

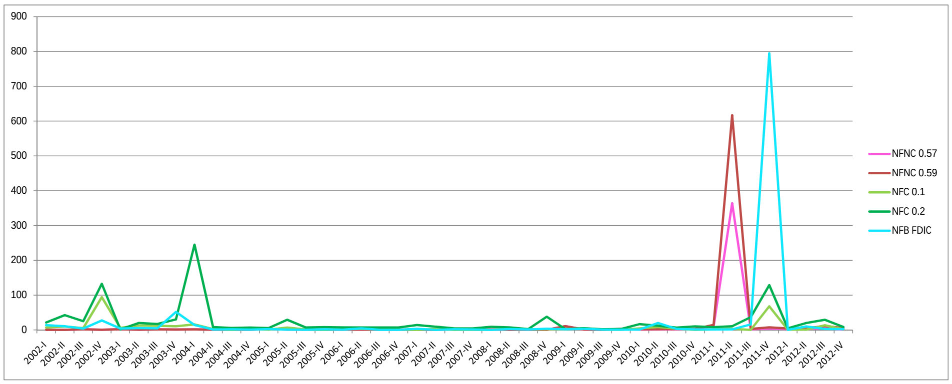

Figure 4: Comparison of $|b_{22}|$ for NFNC with E/A ratio 0.57 and 0.59, $|b_{33}|$ for NFC with E/A ratio 0.1 and 0.2, and $|b_{88}|$ for combined NFB with 'bad loans' from FDIC data. It shows that NFNC becomes unstable before NFC, which makes sense because many small and medium enterprises, that are more vulnerable than corporations, belong to NFNC category.

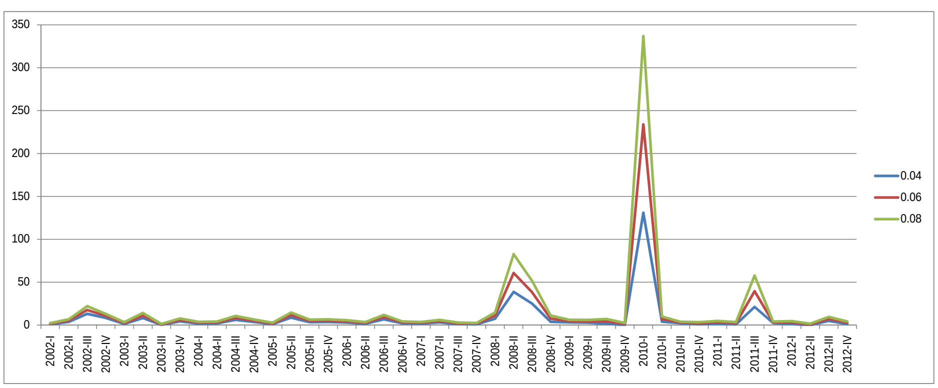

Figure 5: The market instability indicator $|b_{44}|$ for FB with $\sum_{i\neq 4} a_{i4}$ calculated from E/A ratio 0.04, 0.6, and 0.08.

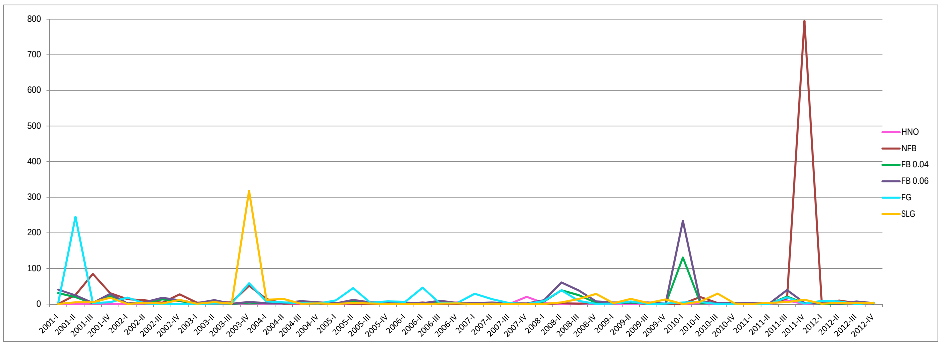

Figure 6: The market instability indicator for five domestic agents (combined NFB for NFNC and NFC) from 2001 Q1 to 2012 Q4. Two capital requirement ratios for FB are used.

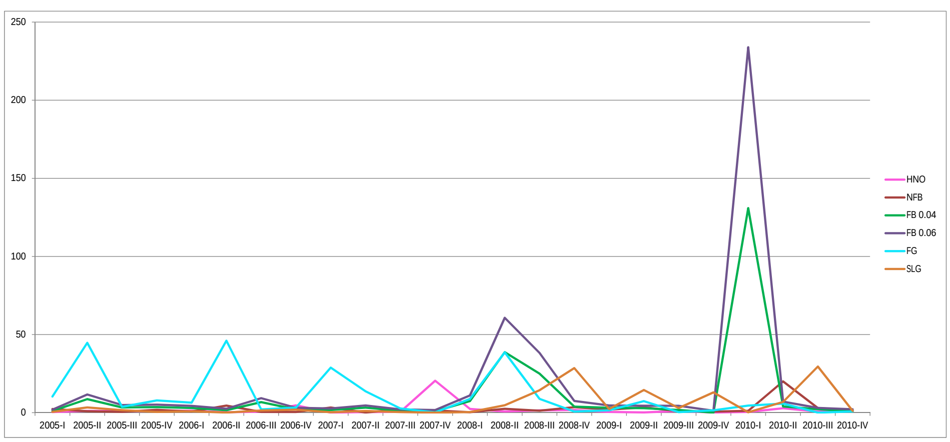

Figure 7: Figure 6 for a shorter time frame. Between 2007 Q3 and 2009 Q1, we can see that the indicators for HNO, FB and FG, and SLG peak in that order, which suggests contagion of financial instability. Combined NFB peaks with two quarters' lag. IMA data show that cash inflows and deleveraging accelerate during 2009 Q4 for both NFNC and NFC. This instability hike would be mostly due to this 'hoarding cash.'

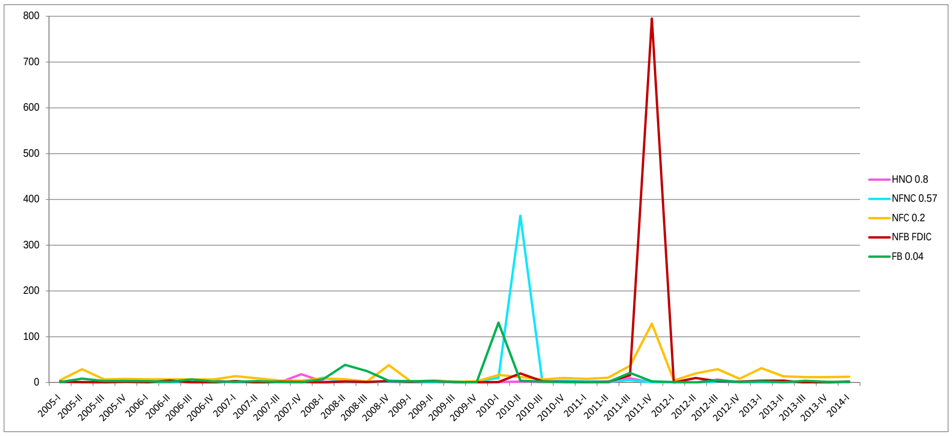

Figure 8: The market instability indicators for domestic private sector agents from 2005 Q1 to date. Selected indicators for separate NFNC and NFC as well as combined NFB are shown for comparison purpose.

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