Extreme Dependence Modelling in Energy Markets using Sibuya-type Copulas

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The evidence of joint extreme movements when energy markets go up can be possibly explained by the simultaneous impact of increasing economic and political activities in different geographic regions. In a first step of analysis, several single equation models are applied to account for autocorrelation and volatility clustering in marginals. Their variability is huge and depend on many factors, but state-space time series and factor models are exploited usually.

In general, the error terms are assumed to be uncorrelated (i.e. independent and identically distributed). The empirical analysis shows that this theoretical belief is frequently violated in practice, especially before/after crisis periods. Recently, in a second step, professionals select an appropriate copula family characterizing the joint dynamics between the time series. Since the copula is a function of marginal distribution (error terms), an adequate modelling of individual time series is crucial for estimating the dependence structure between underlying commodities, say.

In this talk, I am suggesting a general three-step procedure for analysis of extreme dependence in energy markets as follows.

Step 1. Use the General Autoregressive Score (GAS) model for time varying parameters, (see Creal, Koopman and Lucas (2013), *J. of Appl. Economics* 28, 777-795 and recent developments in *http://gasmodel.com*);

Step 2. Select an appropriate Sibuya-type copula, (introduced by Pinto (2014), PhD Thesis, IME-USP);

Step 3. Perform corresponding Monte Carlo simulations.

GAS models postulate different dynamics for volatilities from fat-tailed distributions. Particular score function choices reduce GAS model to well known classical time series.

The Sibuya-type copulas are generated by relation

$$r_1(x,y) + r_2(x,y) = a_0 + a_1 x + a_2 y, \quad (x,y) \in [0,\infty)^2,$$

where $(r_1(x,y), r_2(x,y))$ is vector of (conditional) hazard components and $a_0, a_1, a_2 \geq 0$. The Sibuya-copula class is large, including absolutely continuous or singular copulas, which are symmetric or asymmetric, extreme value, positive quadrant dependent or not.

Our empirical investigation focuses on the dependence between crude oil and natural gas prices. The data set covers quotes of the Light Sweet Crude Oil Futures and the Henry Hub Natural Gas Futures on NYMEX between July 2, 2007 and July 2, 2010.