## Functional Itô Calculus, Path-dependence and the Computation of Greeks

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## Abstract

Dupire's functional Itô calculus provides an alternative approach to the classical Malliavin calculus for the computation of sensitivities, also called Greeks, of path-dependent derivatives prices. In this paper, we introduce a measure of pathdependence of functionals within the functional Itô calculus framework. Namely, we consider the Lie bracket of the functional space and time derivatives, which we use to classify functionals according to their degree of path-dependence. We then revisit the problem of efficient numerical computation of Greeks for pathdependent European derivatives using integration by parts techniques. Special attention is paid to path-dependent functionals with zero Lie bracket, called weakly path-dependent functionals in our classification. We then derive the well-known weighted-expectation formulas for their Greeks, that was first derived using Malliavin calculus. In the more general case of fully path-dependent functionals, we are able to analyze the effect of our path-dependence measure into the Greeks computation. Numerical examples are also provided.