On the complexity of the steepest descent algorithm for minimizing convex quadratic functions

Clovis C. Gonzaga
Federal University of Santa Catarina, Brazil

We discuss the performance of the steepest descent algorithm for minimizing a quadratic function with hessian matrix eigenvalues between $1/C$ and 1. Steepest descent methods differ exclusively on the choice of the step length at each iteration. We develop a scheme for choosing the step lengths with the following result: the number K of iterations needed to reduce the objective function, the gradient norm and the distance to the optimal solution by a factor ϵ is bounded by $K \leq \sqrt{C}\log(1/\epsilon)$.

This is similar to the bound given by Krylov space methods, and better than the bound derived from Kantorovich's analysis.