

(Multi)singular hyperbolic structures

Christian Bonatti ¹

¹ Université de Bourgogne

Resumo/Abstract:

A vector field X is called a star flow if every periodic orbit of every C^1 -close vector field Y is hyperbolic. With Andriana da Luz, we built a star flow on a 5-manifold with two singular points of distinct indices which are robustly in the same chain recurrence class. The interest of such example is that it forces us to make compatible the hyperbolicity of the regular orbits (which essentially lies on the normal bundle of X) with the hyperbolicity of the singular points, (which lies on the tangent bundle). When all singular points have the same index, Gan Wen and Yi proved that the star property is equivalent to “singular hyperbolicity” that is, the existence of a dominated splitting for the flow which is uniformly contracting in one bundle and uniformly sectionally area expanding in the other. Our example does not admit any splitting invariant under the flow, and in particular, no dominated splitting. Hyperbolicity lies elsewhere. The case of star flows is just an example among all the situations where we will need to adapt the weak notions of hyperbolicity (partial hyperbolicity, volume hyperbolicity,...) to vector fields presenting recurrent orbits visiting singularities. With Adriana da Luz, we propose a general definition making compatible the notions of hyperbolicity along regular orbits and on singularities.