

A Compactness Result for Energy-minimizing Harmonic Maps with Rough Domain Metric *

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Abstract:

In 1996, Shi [6] generalized the ϵ -regularity theorem of Schoen and Uhlenbeck [5] to energy-minimizing harmonic maps from a domain equipped with a Riemannian metric of class L^∞ . In the present work we prove a compactness result for such energy-minimizing maps. As an application, we combine our result with Shi's theorem to give an improved bound on the Hausdorff dimension of the singular set, assuming that the map has bounded energy at all scales. This last assumption can be removed when the target manifold is simply-connected thanks to a universal energy bound due to Hardt, Kinderlehrer and Lin [4].

References

- [1] Giuseppe Buttazzo and Gianni Dal Maso. Γ -limits of integral functionals. *J. Analyse Math.*, 37:145{185, 1980.
- [2] Gianni Dal Maso. *An Introduction to Γ -Convergence*. Birkhauser, Boston, 1994.
- [3] Herbert Federer. *Geometric measure theory*. Springer-Verlag New York Inc., New York, 1969.
- [4] Robert Hardt, David Kinderlehrer, and Fang-Hua Lin. Stable defects of minimizers of constrained variational principles. *Ann. Inst. H. Poincaré Anal. Non Linéaire*, 5(4):297{322, 1988.
- [5] Richard Schoen and Karen Uhlenbeck. A regularity theory for harmonic maps. *J. Differential Geom.*, 17(2):307{335, 1982.
- [6] Yuguang Shi. A partial regularity result of harmonic maps from manifolds with bounded measurable Riemannian metrics. *Comm. Anal. Geom.*, 4(1-2):121{128, 1996.
- [7] Leon Simon. *Theorems on regularity and singularity of energy minimizing maps*. Birkhauser Verlag, Basel, 1996.