Regularity issues in semilinear Schrödinger and heat equations

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Resumo/Abstract:
We consider the Cauchy problem for the semilinear heat and Schrödinger equations on R^N, with the nonlinearity $f(u) = \lambda |u|^\alpha u$.

We first show that low regularity of f (i.e., $\alpha > 0$ but small) limits the regularity of any possible solution for a certain class of smooth initial data. We employ two different methods. On the one hand, we consider the semilinear equation as a perturbation of the ODE $w_t = f(w)$. This yields in particular an optimal regularity result for the semilinear heat equation in Hölder spaces. In addition, this approach yields ill-posedness results for NLS in certain H^s spaces, which depend on the smallness of α rather than the scaling properties of the equation. Our second method is to consider the semilinear equation as a perturbation of the linear equation via Duhamel's formula. This yields in particular that if α is sufficiently small and N sufficiently large, then the nonlinear heat equation is ill-posed in $H^s(R^N)$ for all $s \geq 0$.

The possible singularity of the nonlinearity f is only at 0. Therefore, regularity issues can be avoided for solutions that never vanish. Such solutions can easily be constructed for the nonlinear heat equation by using the maximum principle. It turns out that one can also construct such solutions for NLS. It follows in particular that for every $\alpha > 0$ and $\lambda \in C$ there is a class of initial values for which there exists a local solution of NLS. Moreover for every $\alpha > \frac{2}{N}$, there is a class of (arbitrarily large) initial values for which there exists a global solution that scatters as $t \to \infty$.

The results presented in this talk are based on joint works with F. Dickstein, I. Naumkin and F. Weissler.