

On the inhomogeneous nonlinear Schrödinger equation

Carlos M Guzmán¹, MY CO-AUTHOR'S Luiz G Farah²

^{1,2} Federal University of Minas Gerais

Resumo/Abstract:

The purpose of this work is to study the initial value problem for the inhomogeneous nonlinear Schrödinger equation (INLS)

$$\begin{cases} i\partial_t u + \Delta u + \lambda|x|^{-b}|u|^\alpha u = 0, & t \in \mathbb{R}, x \in \mathbb{R}^N, \\ u(0, x) = u_0(x), \end{cases} \quad (1)$$

where $u = u(t, x)$ is a complex-valued function in space-time $\mathbb{R} \times \mathbb{R}^N$, $\lambda = \pm 1$ and $\alpha, b > 0$.

We obtain local and global results for the initial data in $H^s(\mathbb{R}^N)$, with $0 \leq s \leq 1$. Moreover, we prove scattering ($\lambda = 1$) for the radial data in $H^1(\mathbb{R}^N)$. To this end, for the first one we use the contraction mapping principle based on the Strichartz estimates and to show scattering we use the ideas introduced by Kenig-Merle [3] in their study of the energy-critical NLS, Holmer-Roudenko [2] for the 3D cubic NLS and Fang-Xie-Cazenave [1] for the general case (dimension N).

References

- [1] D. FANG, J. XIE, AND T. CAZENAVE, *Scattering for the focusing energy-subcritical nonlinear Schrödinger equation*, Sci. China Math, 54(10):2037–2062, 2011.
- [2] J. HOLMER, S. ROUDENKO, *A sharp condition for scattering of the radial 3D cubic nonlinear Schrödinger equation*, Comm. Math Phys, 435–467, 2008.
- [3] C. E. KENIG, F. MERLE, *Global well-posedness, scattering, and blow-up for the energy-critical focusing nonlinear Schrödinger equation in the radial case*, J. Invent. Math, 166: 645–675, 2006.