Anne Bertrand. Boyle and Handelman conjecture, primitive matrices in $\mathbb{N}$ and Perron numbers.

Abstract. A Perron number is an algebraic integer $\lambda > 1$ whose conjugates μ_i all verify $|\mu_i| < \lambda$; a square matrix A is primitive if its entries are ≥ 0 and if for some k all the entries of A^k are > 0. A primitive matrix whose entries belong to $\mathbb{N}$ admits a dominating eigenvalue λ that is a Perron number, and all other eigenvalues have absolute value $< \lambda$.

Lind proved that a Perron number λ is always the dominating eigenvalue of a primitive matrix on $\mathbb{N}$. But the order of the matrix can be greater than the degree of λ and eigenvalues that are not conjugates of λ can appear. Under a special condition of Boyle and Handelman, Kim, Ormes and Roush proved that there is a primitive matrix on $\mathbb{N}$ whose eigenvalues are λ, its conjugates and possibly few zeros. The size of matrix was still unknown.

We prove that given a Perron number λ of degree d there exists a number r_0 such that if $r \geq r_0$ then there is a primitive matrix of order d on $\mathbb{N}$ whose eigenvalues are only λ and its conjugates.

We also give a proof of Lind's theorem based on prefix codes. We shall also take up the following problem: let $\log \beta$ be the entropy of a symbolic dynamical system, and let w_n be the number of n–strings of the system. Can we determine a system minimizing w_n (for all n or asymptotically)? We conjecture that in many cases the so-called β–shift minimizes w_n. We know very few on this subject.
