An Introduction to mathematical modelling of biophysical phenomena with PDEs

Level : M2 (master)
Duration : 32 hours (during the summer semester at IMPA)

Since several years, mathematical modelling of biological phenomena has become of a great interest. In this framework, partial differential equations (PDE) are a usefull tools to describe observed phenomena.

The aim of this course is to present to master's student several PDE models that describe different biophysical phenomena and their analysis. The main parts that will be presented in this course are the following :

1. **Introduction :** introduction of several examples of ODE and PDE models of biophysical phenomena.
2. **Population dynamics :** analysis of systems of ODE used in population dynamics and applications.
3. **Transport equations in biology :** weak solutions of transport equations, method of characteristics, application in biology.
4. **Parabolic equation in biology :** properties of weak solutions, examples.
5. **Traveling waves :** modelling of invasive front, traveling waves for the Fisher/KPP and Allan-Cahn equations, traveling pulse and pike.
6. **Blow-up of solutions.**
7. **The Fokker-Planck equation** and the scattering equation.
8. **Linear and Turing instabilities :** application to the formation of patterns in nature.

Bibliogaphy :

- H. Brézis, Analyse fonctionnelle, théorie et applications. Masson, 1983.
- L.C. Evans, Partial Differential Equations, Graduate Studies in Mathematics, Vol 19, American Mathematical Society (1998).
- J.D. Murray, Mathematical biology, Vol 1 and 2, Second edition. Springer, 2002.
- B. Perthame, Transport equations arising in biology, L.N. Series 'Frontiers in mathematics', Birkhauser, 2007.
- B. Perthame, Growth, reaction, movement and diffusion from biology, M2 course, UPMC.