Sharp nonasymptotic bounds on the norm of random matrices with independent entries

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Abstract:
We obtain nonasymptotic bounds on the spectral norm of random matrices with independent entries that improve significantly on earlier results. If X is the n×n symmetric matrix with. If X is the n×n symmetric matrix with $X_{ij} \sim N(0, b_{ij}^2)$, e show that $\mathbf{E}\|X\| \lesssim \max_i \sqrt{\sum_j b_{ij}^2} + \max_{ij} |b_{ij}| \sqrt{\log n}$.

This bound is optimal in the sense that a matching lower bound holds under mild assumptions, and the constants are sufficiently sharp that we can often capture the precise edge of the spectrum. Analogous results are obtained for rectangular matrices and for more general subgaussian or heavy-tailed distributions of the entries, and we derive tail bounds in addition to bounds on the expected norm. The proofs are based on a combination of the moment method and geometric functional analysis techniques. As an application, we show that our bounds immediately yield the correct phase transition behavior of the spectral edge of random band matrices and of sparse Wigner matrices. We also recover a result of Seginer on the norm of Rademacher matrices.