Hyperbolic Geometry and Minimal Surfaces

IMPA, Rio de Janeiro, from 01/04 to 01/10.

Abstracts

Monday, Jan. 5

Hyam Rubinstein (Melbourne)
Title: Shortest geodesics and minimal surfaces.
Abstract: Tubes about short geodesics can be used as barriers to produce multiple minimal surfaces in isotopy classes and minimal surfaces with unusual topological properties. In work of Minsky, Namazi, Souto etc, global properties of hyperbolic 3-manifolds are given. An interesting challenge is to relate this to behaviour of minimal surfaces. Tubes give a first step in this direction.

Regina Rotman (Toronto)
Title: Quantitative homotopy theory and the lengths of geodesics on Riemannian manifolds
Abstract: Let M be a closed Riemannian manifold. There are numerous results that establish the existence of various minimal objects on M, such as periodic geodesics, minimal surfaces, or geodesic nets. We will present some effective versions of these existence theorems.

For example, we will present diameter upper bounds for the lengths of three shortest simple periodic geodesics on a Riemannian 2-sphere, which can be interpreted as an effective version of the existence theorem of Lusternik and Schnirelmann. (Joint with Y. Liokumovich and A. Nabutovsky).

Finding upper bounds for the size of smallest stationary objects is closely related with construction of "optimal" homotopies. We will show that if M is a closed surface of diameter d (with or without boundary), then any simple closed curve on M that can be contracted to a point over free loops of length less than L, can be contracted over based loops of length at most 3L+2d. (Joint with G. Chambers).

Ivaldo Nunes (São Luís)
Title: Rigidity of area-minimizing hyperbolic surfaces in three-manifolds
Abstract: In this talk, we prove a local rigidity result for cylinders over hyperbolic surfaces. More precisely, we obtain a lower bound for the area of area-minimizing minimal surfaces of genus greater than 1 inside three-manifolds with scalar curvature bounded below by a negative constant. Moreover, if equality holds then the induced metric on the surface is the hyperbolic metric and locally the ambient space splits as a cylinder over the surface.

José Espinar (IMPA)
Title: Escobar type Theorem for fully nonlinear Yamabe problem with boundary.
Abstract: In this talk we extend Escobar's classification result of the Yamabe Problem with boundary to elliptic fully nonlinear conformal equations on certain subdomains of the sphere with prescribed constant mean curvature on its boundary. Such subdomains are the hemisphere (or a geodesic ball on $\mathbb{S}^n$) with prescribed constant mean curvature on its boundary, and annular domains with minimal boundary.

Joel Hass (Davis)
Title: Minimal surfaces in 3-manifolds that fiber over S^1
Abstract: A 3-manifold that fibers over the circle admits a metric in which each leaf is a minimal surface. When the monodromy of the bundle is pseudo-Anosov, the manifold admits a hyperbolic metric. We show that for some manifolds these two metrics cannot coincide. Some hyperbolic 3-manifolds that fiber over the circle cannot be fibered by minimal fibers. This was joint work with Bill Thurston.

Tuesday, Jan. 6

Dave Gabai (Princeton)
Title: On the classification of Heegaard splittings
Abstract: We discuss how geometric methods can be used to obtain an algorithm to enumerate without duplication the irreducible Heegaard splittings of closed non Haken 3-manifolds. Joint work with Toby Colding and Dan Ketover.

Zeno Huang (CUNY)
Title: Closed minimal surfaces in hyperbolic three-manifolds
Abstract: Closed minimal surfaces do not exist in Euclidean three space, nor in hyperbolic three-space, but they are abundant in hyperbolic three-manifolds. Following a pioneer work of Uhlenbeck, we study the space of closed minimal surfaces in hyperbolic three-manifolds. We will present some results of uniqueness, and also results of non-uniqueness for certain minimal surfaces.

Frank Pacard (École Polytechnique)
Title: The role of Green's function in doubling constructions for minimal and constant mean curvature surfaces
Abstract: TBA

Moon Duchin (Tufts)
Title: Hyperbolicity from a statistical viewpoint
Abstract: Can hyperbolicity of a metric space be efficiently certified with random measurements? To what extent are random groups or manifolds hyperbolic? And what are the properties of typical hyperbolic objects? I will survey some research directions and describe some recent joint work.

Hugo Parlier (Fribourg)
Title: Combinatorial moduli spaces
Abstract: Combinatorial spaces, often related to simple closed curves on surfaces, have been used in different ways to understand Teichmüller spaces, mapping class groups and moduli spaces. More specifically, the curve and pants graphs have been helpful tools to understand geometric properties of Teichmüller spaces with its different metrics and the mapping class group. Flip graphs are other examples of useful combinatorial spaces. The vertices of these graphs are isotopy classes of triangulations and two triangulations share an edge if they are related by a flip (or equivalently differ by a single arc). Flip graphs are also conveniently quasi-isometric to the underlying mapping class groups. The flip graph of a polygon, although finite, has been particularly well studied, most famously by Sleator, Tarjan and Thurston who studied its diameter. In another piece of work, they studied the diameters of flip graphs of punctured spheres (this time up to the action of their mapping class groups).

The talk will be about some recent results about the geometry of these graphs for more general surfaces. Part of the talk will be about joint work with V. Disarlo and another part joint with L. Pournin.

Wednesday, Jan. 7

Michael Anderson (Stonybrook)
Title: Handlebodies with constant curvature metrics and minimal surface boundary
Abstract: We study the moduli space of constant curvature metrics g on a 3-d handlebody with boundary having mean curvature 0, so minimal surface boundary (or more generally CMC boundary). This is a generalization of Alexandrov immersed minimal surfaces in 3-d space forms.

We prove that this moduli space is a smooth manifold, locally diffeomorphic to the Teichmuller space of the boundary surface, when the genus of the boundary is at least 2. We conjecture that the spaces are in fact diffeomorphic (on each component).

This result is false per se for genus 1 boundaries, but the method of proof gives rise to a new proof of Brendle's solution of the Lawson conjecture on embedded minimal tori in S^3.

The talk will discuss the context and basic ideas of the proof. We hope to discuss relations and/or questions with hyperbolic 3-manifolds.

Laurent Mazet (Paris-Est)
Title: Minimal surfaces in finite volume hyperbolic 3-manifolds
Abstract: We are interested in the geometry of minimal surfaces when the ambiant space is a finite volume hyperbolic 3-manifold. The talk will deal about the asymptotic behaviour of minimal surfaces of finite topology and a "finite total curvature theorem". This is a joint work with P. Collin, L. Hauswirth and H. Rosenberg.

Toby Colding (MIT)
Title: TBA

Thursday, Jan. 8

Fernando Codá Marques (Princeton)
Title: Min-max theory and applications I

André Neves (Imperial College)
Title: Min-max theory and applications II

Abstract: In these two talks we will give an overview of the min-max theory for minimal surfaces as developed by Almgren and Pitts, and describe the most recent developments

Stephane Sabourau (Paris-Est)
Title: Sweepout estimates on Riemannian manifolds
Abstract: We will present min-max estimates on the volume width of Riemannian surfaces and nonnegatively curved Riemannian manifolds. These estimates lead to geometric inequalities relating the volume of minimal hypersurfaces to the volume of the whole manifold.

Stephen Kleene (Brown)
Title: Non-compactness of Moduli spaces of finite topology embedded minimal surfaces
Abstract: I will discuss recent work in which singular perturbation methods are applied to show non-compactness of the moduli space M(4, g) of finite topology minimal surfaces with four ends and high genus g. Additionally, I will outline how I expect the technique to generalize to the space M(k, g). This is joint work with Niels Martin Moller.

Alan Reid (Austin)
Title: Determining hyperbolic 3-manifolds by geometric spectra.
Abstract: This talk will discuss to what extent closed hyperbolic 3-manifolds can be determined by various geometric spectra such as length spectra, and the set of pi_1-injective surfaces.

Friday, Jan. 9

William Meeks (Amherst)
Title: Minimal and CMC surfaces in homogeneous 3-manifolds
Abstract: I will talk about recent progress in the theory of these surfaces, mostly concerning joint work with Joauqin Perez and Giuseppe Tinaglia.

Marc Lackenby (Oxford)
Title: Cusp volumes of hyperbolic alternating knots
Abstract: I will outline my proof, with Jessica Purcell, that there is a uniform lower bound on the cusp density of hyperbolic alternating knots. The argument uses a range of geometric and topological techniques to establish the existence of many short geodesic arcs in the checkerboard surfaces, when they are placed in pleated form.

Steve Kerckhoff(Stanford)
Title: Boundary value problems and deformations of hyperbolic 3-manifolds
Abstract: TBA

Laurent Hauswirth (Paris-Est)
Title: Spectral curves, harmonic maps and minimal surfaces
Abstract: N. Hitchin introduced spectral curves representation to describe harmonic map from a 2-torus into S(2) or S(3). Considering inifinite covering annuli of tori, we will explain the space of periodic harmonic maps induced by deformations of spectral curves and how this describe the geometry of properly embedded minimal surfaces in S(2)xR.

Jeff Brock (Brown)
Title: Bounded geometry and uniform models for hyperbolic 3-manifolds
Abstract: In this talk I will describe joint work with Minsky, Namazi, and Souto describing a general framework for understanding the geometry of hyperbolic 3-manifolds of 'bounded type.' As an application I will describe models for Heegaard splittings of bounded type and outline a converse that provides models for splittings of bounded geometry.