**INTERPORE**

1ˢᵗ IMPA-InterPore
# Workshop on Porous Media



**IMPA, Rio de Janeiro, Brazil**
**October 19-21, 2014**

# Program
# & Abstracts

# 1st IMPA-INTERPORE

# Workshop on Porous Media

**IMPA, Rio de Janeiro, Brazil**

**October 19-21, 2014**

*Auditorium 3 - second floor*

**Organizing Committee**

| | |
|---|---|
| Alexei A. Mailybaev | IMPA, Rio de Janeiro |
| Dan Marchesin | IMPA, Rio de Janeiro |
| Aparecido J. de Souza | Universidade Federal de Campina Grande |

## SCIENTIFIC PROGRAM

### SUNDAY 19/OCT

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|---|---|
| **12:30-14:00** | **REGISTRATION / BRUNCH** |
| **14:00-14:10** | **OPENING** |
| | Session chair: Hans Bruining |
| **14:10-15:10** | On the scale dependency of nonequilibrium flow and contaminant transport processes<br>MARTINUS TH. VAN GENUCHTEN (UFRJ, Rio de Janeiro) |
| 15:10-15:40 | Hysteretic enhancement of carbon dioxide trapping in deep aquifers<br>FREDERICO FURTADO (University of Wyoming, USA) |
| **15:40-16:00** | **COFFEE BREAK** |
| | Session chair: Adolfo P. Pires |
| 16:00-16:30 | Combustion for enhanced recovery of light oil at medium pressures<br>NEGAR K. GARGAR (TU Delft, The Netherlands) |
| 16:30-17:00 | Recovering porous density distribution of activated carbon samples<br>AMAURY C. ALVAREZ (Oceanology Institute, C. Habana, Cuba) |
| 17:00-17:30 | A spectral decomposition strategy for decoupling an integro-differential model of expansive porous media<br>SAULO P. OLIVEIRA (Universidade Federal do Paraná, Curitiba) |

## MONDAY 20/OCT

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|---|---|
| | Session chair: Frederico Furtado |
| **09:20-10:00** | Coupled hydro-geomechanical computational models of pre-salt reservoirs<br>MARCIO A. MURAD (LNCC, Petrópolis) |
| **10:00-10:30** | Geochemical modeling of water flooding oil in calcite reservoirs at high carbon dioxide pressure<br>HANS BRUINING (TU Delft, The Netherlands) |
| **10:30-10:50** | **COFFEE BREAK** |
| | Session chair: Grigori Chapiro |
| **10:50-11:20** | Wave sequences for solid fuel adiabatic in-situ combustion in porous media<br>APARECIDO J. DE SOUZA (Universidade Federal de Campina Grande) |
| **11:20-11:50** | Combustion fronts in a porous medium with two parallel layers<br>JESUS CARLOS DA MOTA (Universidade Federal de Goiás) |
| **11:50-12:20** | Unusual thermal effects in Riemann solutions for two-phase flow in porous media<br>JULIO DANIEL SILVA (University of Calgary, Canada) |
| **12:20-14:00** | **LUNCH** |
| | Session chair: Dan Marchesin |
| **14:00-15:00** | Front tracking for petroleum reservoirs: nonlinearity vs. dispersion and the role of heterogeneity and scaleup<br>JAMES GLIMM (Stony Brook University, USA) |
| **15:00-15:30** | Recent developments in the approximation of high-contrast flow problems<br>JUAN GALVIS (Universidad Nacional de Colombia) |
| **15:30-16:00** | Using a partially-linear Koval's type model to study the flow of supercritical $CO_2$, water and oil in porous media: pressure-driven and buoyancy-driven flows<br>PANTERS RODRÍGUEZ-BERMÚDEZ (Universidade Federal Fluminense) |
| **16:00-16:20** | **COFFEE BREAK** |
| | Session Chair: Fabio P. dos Santos |
| **16:20-16:50** | Predictive modeling of subsurface flows<br>FELIPE PEREIRA (The University of Texas at Dallas, USA) |
| **16:50-17:20** | Numerical simulation of reservoir-wellbore coupled flow<br>ADOLFO P. PIRES (Universidade Estadual do Norte Fluminense) |
| **17:20-17:50** | Computational methods for simulation of porous media transport phenomena<br>EDUARDO ABREU (IMECC/UNICAMP, Campinas) |
| **17:50-18:20** | **BRAZILIAN CHAPTER OF INTERPORE: KICKOFF MEETING** |
| **18:20** | **WORKSHOP COCKTAIL** |

## TUESDAY 21/OCT

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|---|---|
| | Session chair: Juan Galvis |
| **09:20-10:00** | Advances in mathematical modeling of turbulent transport in porous media<br>MARCELO J.S. DE LEMOS (ITA, São José dos Campos) |
| **10:00-10:30** | The Mathematical Theory of Three-Phase Porous Medium Flow<br>BRADLEY PLOHR (Los Alamos National Laboratory, USA) |
| **10:30-11:20** | **POSTERS / COFFEE BREAK**<br><br>Asymptotic approximation analysis for counterflow combustion in porous media<br>GRIGORI CHAPIRO (UFJF, Juiz de Fora)<br><br>The random Riemann solution for the Buckley-Leverett equation<br>DUILIO CONCEIÇÃO (UFRRJ, Rio de Janeiro)<br><br>The wave curve method: numerical implementation<br>DAN MARCHESIN (IMPA, Rio de Janeiro)<br><br>Semi-analytical solution for microwave assisted water injection in porous media<br>PAVEL Z. SEJAS PAZ (UFJF, Juiz de Fora)<br><br>Numerical solution of in-situ combustion model using finite element method combined with nonlinear complementarity algorithm<br>WESLLEY S. PEREIRA (UFJF, Juiz de Fora) |
| | Session chair: Max Kokubun |
| **11:20-11:50** | Investigation for the recovery of medium-heavy oil by gas injection<br>PACELLI L.J. ZITHA (TU Delft, The Netherlands) |
| **11:50-12:20** | Air injection rate effects in light oil recovery<br>FABIO PEREIRA DOS SANTOS (IMPA, Rio de Janeiro) |
| **12:20-14:00** | **LUNCH** |
| | Session chair: Amaury C. Alvarez |
| **14:00-14:30** | On porous media flows for three components and two phases<br>VITOR M.M. MATOS (Universidade do Porto, Portugal) |
| **14:30-15:00** | Incorporating topography into soil hydraulic parameter scaling algorithms<br>RAGHAVENDRA B. JANA (King Abdullah University of Science and Technology, KSA) |
| **15:00-15:30** | How to predict soil water holding capacity using physically based equations?<br>WENCESLAU TEIXEIRA (Embrapa Solos, Rio de Janeiro) |
| **15:30-16:00** | Singularity in the combustion wave profile for porous media with liquid fuels<br>ALEXEI A. MAILYBAEV (IMPA, Rio de Janeiro) |
| **16:00-16:10** | **CLOSING** |

# Abstracts

# *Computational methods for simulation of porous media transport phenomena*

## *Eduardo Abreu*

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The mathematical modelling and simulation of porous media flow problems is complex and therefore several relevant prototype models have been proposed for a qualitative study for understanding multiphase immiscible displacement problems in porous media [3,4,5,6,7] such as the scalar case of immiscible two-phase flow (the classical Buckley-Leverett water/oil and water-gas problems) and the system case to simultaneous immiscible three-phase flow (the nonclassical oil/water/gas flow problems). Those are a set of intricate and highly nonlinear governing mixed hyperbolic-parabolic systems of partial differential equations leading to the development of novel studies for theory, computation methods and qualitative behavior of their solutions (e.g., [3,4,5,6,7]). We would like to report on the computational method intended for simulating three-phase immiscible incompressible flow system in porous medium in two space dimensions [1,2], taking the form of a nonlinear mixed hyperbolicparabolic convection-diffusion system with spatially varying flux functions under combined convective, capillary and gravity effects. Our new computational method [1,2] is an operator-splitting procedure for decoupling the hyperbolicparabolic three-phase flow equations with mixed discretization methods, locally conservative by construction, leading to purely hyperbolic, parabolic and elliptic subproblems. The formulation reported here is designed to handle discontinuous capillary pressures in diffusion and to take into account the explicit spatially discontinuous flux functions in convection. A set of representative numerical examples is discussed to assess the effectiveness and the reliability of the new approach [1,2].

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# Recovering porous density distribution of activated carbon samples

### *A. C. Alvarez*
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### *N. Passé-Coutrin, S. Gaspard*
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Synthesis of porous material with remarkable properties to clean polluted efficiently water is a current challenge. In such sense activated carbon can be obtained from various biomass including marine plants. The characterization of porous surface gives the measure of adsorptive properties of such porous media. Gas adsorption of gases such as $N_2$ is well known technique for textural characterization of porous surface. In such sense different experimental isotherm are obtained which enables the determination of porous density distribution and porous analysis. In this paper we proposed two recovered methods to estimate the porous density distribution based on images of porous surface and experimental isotherm. The recovered procedures consist in solving a first kind integral equation and images processing. Both method show accuracy in the porous characterization and give similar results. The method deals with the difficulty that experimental Kernel gives a severe ill-posed problem due to the eigenvalues go to zero rapidly. The images processing procedure is a less spencer experimental method that give additional information to obtain the regularized solution.

# Geochemical modeling of water flooding oil in calcite reservoirs at high carbon dioxide pressure

### *Hans Bruining* (Delft University of Technology, The Netherlands)

### **Dan Marchesin (**IMPA, Brazil)

### **Rouhi Farajzadeh (**Shell Global Solutions International)

### **Wanderson Lambert** (UFRRJ, Rio de Janeiro)

### *Vitor Matos* (Universidade do Porto, Portugal)

### **Hua Guo, Hamidreza Salimi** (PanTerra Geoconsultants BV)

We are interested in low salinity water flooding of light oil with dissolved carbon dioxide at high pressure in a calcite reservoir, representative for the presal formations off the coast of Brazil. We study this process in a 1-D setting in an oleic phase that includes carbon dioxide and an aqueous phase that contains all the ionic substances. By assuming chemical equilibrium we can describe the motion of all dissolved compounds in a limited number of transport equations of master species. Active sites on calcite and oil are in chemical equilibrium with the ions in the solution to form surface complexes. Geochemistry can be accounted for by (a) use of geochemical software (PHREEQC) to create look-up tables with regression (b) simultaneous solution of all relevant equations, which however, requires some understanding of geochemistry.

The concentration of the surface complexes determines the charge (PHREEQC) on the calcite and the oil surfaces. If the charges on the surfaces have the same sign, they repel each other and allow a water film to be formed in between. On the other hand, when they have opposite charges, they attract each other and the water film will be squeezed out. A system with water films can be expected to be water-wet. The wetting properties determine both the behavior of the relative permeabilities and capillary pressure. We expect that this formalism is useful for determining the oil recovery when low salinity water is injected as a secondary recovery technique.

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## Asymptotic approximation analysis for counterflow combustion in porous media

### Grigori Chapiro

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Air injection and in-situ combustion have long been considered as potential techniques for displacement and recovery of medium and heavy oil. They utilize heavy and immobile oil components as fuel producing heat and improving the recovery of upgraded crude oil.

We consider a porous rock cylinder with a homogeneously distributed solid fuel, initially filled with air that is injected at constant rate on the left end of the cylinder. We assume that combustion starts at the production end and propagates upstream towards the injection end. A bimolecular reaction is assumed to take place between the injected oxygen and the solid fuel, hence the region of reaction behaves as a source of heat as well as a sink for the oxygen and the fuel. We neglect air compressibility and heat losses.

Assuming that the combustion front has a traveling wave profile, we analyze the possible wave sequences present for the counter-flow combustion. Besides the analysis of the wave sequences we apply the asymptotic expansion technique for ordinary differential equations to approximate the traveling wave profile of the combustion front. We perform numerical simulations to validate this approximation.

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## The random Riemann solution for the Buckley-Leverett equation

### Duilio Conceição and Wanderson Lambert

UFRRJ, Rio de Janeiro

We solve the Buckley-Leverett equation with random Riemann data and give an analytical expression for the Riemann random solution. Moreover statistical moments, mean, variance and higher moments, are obtained in a integral form. Since the integral can be numerically solved by using quadrature, we can obtain all moments with any desirable precision. We compare the method presented in this paper with the Monte Carlo ones.

# Hysteretic enhancement of carbon dioxide trapping in deep aquifers

*A. Rahunanthan* (Edinboro University of Pennsylvania, USA)

*F. Furtado* (University of Wyoming, USA)

*D. Marchesin* (IMPA, Brazil)

*M. Piri* (University of Wyoming, USA)

The sequestration of supercritical carbon dioxide in saline aquifers has been proposed to mitigate global climate change. An important issue is whether it escapes to the atmosphere; chemical retention is permanent in well-chosen aquifers. The effects of chemical reactions may take time, so short term containment techniques are needed. Taking advantage of $CO_2$ retention due to permeability hysteresis during water imbibition has been suggested. We analyze supercritical $CO_2$ retention in deep aquifers due to relative permeability hysteresis. This retention is predicted to occur in the capillary dominated regime, which predominates in large regions of the aquifer. We use recent steady state measurements of supercritical $CO_2$ and brine relative permeabilities at high pressure and temperature. The usage of a CT scanner, constant flow rate and pressure boundary conditions contribute to the high quality of these measurements.

The analysis is performed by a combination of exact analytic solutions based on fractional flow theory and the method of characteristics, as well as highly accurate modern simulation techniques adapted to take hysteresis into account. This combination is used to analyze the one-dimensional upward flow of fluids, first the injected supercritical $CO_2$, then the injected brine that takes advantage of relative permeability hysteresis to immobilize the $CO_2$. Our analysis finds it advantageous to inject supercritical $CO_2$ at substantial rates before the injection of brine. The injected brine immobilizes the $CO_2$ by means of a hysteretic shock that propagates very quickly, a peculiarity of supercritical $CO_2$ revealed in the experiments, which has no counterpart in oil and is not contemplated in existing commercial simulators. Our analysis can be improved by taking into account supercritical $CO_2$ compressibility, which could conceivably slow the hysteretic shock and reduce its immobilizing efficiency. We believe that considering radial flow away from a well would lead to analogous results through the use of an appropriate similarity variable, but this remains to be done.

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# Recent developments in the approximation of high-contrast flow problems

*Juan Galvis*

Departamento de Matemáticas, Universidad Nacional de Colombia,
Bogotá D.C., Colombia

In typical porous media applications, the permeability is modeled by discontinuous coefficients with large jumps referred to as high-contrast coefficients. In this talk we review some recent results in the numerical approximation of high-contrast flow problems; from results concerning domain decomposition iterative methods to the recently introduced Generalized Multiscale Finite Element Method (GMsFEM) by T. Hou et.al. These methods use local spectral selections to identify important degrees of freedom that have to be taken into account to construct iterative methods and coarse-scale approximations. In particular, we obtain procedures that are robust with respect to the contrast of the coefficients and other parameters that may be present in the model; e.g., nonlinearities and uncertainties. We show some interesting applications of the GMsFEM methodology that illustrate its flexibility and generality. In particular we combine the GMsFEM

framework with other model reduction techniques such as Balance Truncation, Reduced Basis and Empirical Interpolation.

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# Combustion for Enhanced Recovery of Light Oil at Medium Pressures

### Negar Khoshnevis Gargar, Hans Bruining
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### Alexei A. Mailybaev, Dan Marchesin (IMPA, Rio de Janeiro)

Combustion can be used to enhance recovery of heavy, medium or light oil in highly heterogeneous reservoirs. Such broad range of applicability is attained because not only do the high temperatures increase the mobility of viscous oils but also does the high thermal diffusion spread the heat evenly and reduce heterogeneity effects. For the latter reason, combustion is also used for the recovery of light oils. The reaction mechanisms are different for light oils, where vaporization is dominant, whereas for medium non-volatile oils combustion is dominant. We will only consider combustion of medium and light oils. We consider the case in which the oil is directly combusted into small products, for which we use the term medium temperature oil combustion. The model considers evaporation, condensation and reaction with oxygen. The solution consists of three waves, i.e., a thermal wave, a medium temperature oxidation (MTO) wave and a saturation wave separated by constant state regions. The purpose of this paper is to show the effect of diffusion mechanisms, oil composition and water (either initially present or produced by reaction) on the MTO process. The analytical and numerical solutions look qualitatively similar, in spite of the presence of diffusion terms. The numerical results showed that capillary diffusion increased the temperature in the upstream part of the MTO region and decreased the efficiency of oil recovery. However, for realistic conditions, the diffusive processes were shown to have no significant influence on the qualitative behavior of the MTO wave. For light oil mixtures, the solution consists of a thermal wave upstream, a combined vaporization/combustion wave in the middle (with vaporization upstream of combustion) and a saturation wave downstream. For medium mixtures the vaporization/condensation sequence is reversed and vaporization moves ahead of the combustion. Numerical calculations allow to estimate the bifurcation points where the character of the combustion changes from a vaporization-dominated to a combustion-dominated process. We also showed that, when the boiling point of the volatile oil is below or slightly above the boiling point of water, the hot steam region moves upstream of the medium temperature oxidation (MTO), while the volatile oil and steam condense at the same location; it leads to considerable improvement of oil recovery by the MTO wave. Remarkably, the recovery curves (recovery fraction vs. time) depend weakly on the initial water and light oil saturations. If the volatile oil boiling point is much higher than the boiling point of water, the steam region moves downstream of the MTO wave. Numerical calculations suggest the existence of an oil boiling point at which a bifurcation occurs that separates solutions with the steam region upstream or downstream of the combustion zone.

# On the Scale Dependency of Nonequilibrium Flow and Contaminant Transport Processes

*Martinus Th. van Genuchten and Elizabeth M. Pontedeiro*

Departments of Mechanical and Nuclear Engineering, COPPE,
Federal University of Rio de Janeiro, UFRJ, Rio de Janeiro, Brazil

Accurate simulation of nonequilibrium and preferential flow processes in naturally heterogeneous subsurface systems remains a challenge. A range of dual-porosity and dual-permeability formulations are now available for modeling preferential flow in variably-saturated structured media. One computationally very attractive macroscopic modeling approach is to use composite functions for the unsaturated hydraulic conductivity to account for the separate effects of macropores (fractures) and micropores (the soil or rock matrix), and to combine this approach with a mobile-immobile water type physical nonequilibrium formulation for solute transport. Special focus will be on the mass transfer coefficient that is needed to account for solute exchange between the fracture and matrix regions. This coefficient is shown to be strongly dependent upon the scale of the problem being tackled. Literature data confirm its functional dependency on the solute residence time. A similar dependency has long been noted for the longitudinal dispersivity. An example is given illustrating this scale dependency of the mass transfer coefficient for a field-scale heavy metal contamination problem.

# Front Tracking for Petroleum Reservoirs:

## Nonlinearity vs. dispersion and the role of heterogeneity and scaleup

*James Glimm*
Stony Brook University, USA

This talk will address two issues: (a) The nature of front tracking as a numerical algorithm to address discontinuities and steep gradients in numerical simulations, and (b) which aspects of petroleum reservoir simulations argue for or against the importance of this tool.

The nature front tracking. Front tracking, as developed at Stony Brook University, is undergoing a rapid and new development. Here we discuss only the new form of front tracking, not yet totally operational, and some extensions needed for petroleum reservoir modeling. The front is defined by an unstructured triangular grid. Conceptually, the grid is organized into local neighborhood patches, in the spirit of modern differential geometry. Within a single front patch, the coordinates are given in a reference coordinate system, with local polynomial functions. All quantities (front normal, curvature, Jacobeans, etc.) are computed in this coordinate system. The polynomials support up to 6th order accuracy, more than is needed in practice. The front is connected to a fluids code through an API, so that many of its details are hidden from the user and the labor of adaption to a new code is greatly reduced. The most fundamental operations in this API accomplish the propagation of the front, based on calling a velocity field from the host solver. Various support routines check for and repair interface tangling, which may occur during a time step.

A new routine needed for reservoir engineering will be the solution of the elliptic operator associated with Darcy's law. After scaleup, this is defined by a general positive definite matrix, with discontinuous coefficients across a front. We recommend the embedded boundary method (EBM) for solution of this problem. It has already been implemented (in FronTier, but not in the current API package) for delta function source terms at the front and tensor diffusion matrices.

The value of Front Tracking. The value has to do with the occurrence of steep gradienta or discontinuities. In petroleum reservoir modeling, such fronts arise as oil banks and flame fronts. Fronts arise from injection of fluids not otherwise present in the reservoir, but especially they originate and are sustained by flow nonlinearities and they are diminished by dispersive terms in the flow equations. The balance between these nonlinearity and flow dispersion gives rise to a "front width" which should be compared to the mesh length. If the front width is less than or equal to $3\Delta x$, most numerical methods will add significant numerical dissipation to the physical dissipation already in the simulation model and may be a cause of numerical error to be mitigated by front tracking. Tracking for flame fronts has a further advantage of allowing the compression of complex internal chemistry of a front into a single line or surface, as a simplified flame model.

Dissipation is generally the result of averaging small scale effects. Thus not only are the microscopic laws of physics of flow in porous media important but also all aspects of the flow below a grid level. This criteria introduces geological heterogeneities and the need for scale up. Scale up of the Darcy equation remains an elliptic or parabolic equation with modified tensor permeabilities. But scaleup of the Buckley Leverett equation has a more profound effect, adding a new term to the equations. This term is the gradient of a flux, with the flux being a model for the grid level averages of the nonlinear product of v (velocity) and σ (the Buckley Leverett flux), and thus the added flux has the form <v σ> - <v> <σ>, Physically, the most important heterogeneities are thin layers of high or low permeability, and give rise to dispersion of wave fronts, thus countering the nonlinear effects of steepening them. Which of these two competing effects is more important can only be decided on the basis of individual cases.

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## Incorporating topography into soil hydraulic parameter scaling algorithms

### Raghavendra B. Jana
Center for Numerical Porous Media, King Abdullah University of Science and Technology, KSA

### Binayak P. Mohanty
Department of Biological and Agricultural Engineering, Texas A&M University, USA

Scaling of soil hydraulic parameters is urgently necessary for better performance of many hydrological, meteorological, and ecological models. Often the requisite data are measured at a scale inconsistent with the inherent scale at which these models work. Hence, there is a need for scaling schemes which enable one to convert available measured fine resolution data to effective coarser resolution aggregate values or vice versa. Understanding how hydraulic parameters are affected at different scales by the spatial variability of influencing factors such as soil structure and texture, vegetation, and topography is an inherent requirement of efficient scaling schemes. While it is known that connections exist between these factors and the hydraulic parameters, the exact mathematical and/or physical nature of these connections is generally unknown.

Most upscaling efforts for soil hydraulic parameters developed thus far have opted to ignore the effect of topography in their derivation of effective parameter values. This approach, which considers a flat terrain with no lateral flows, is reasonable as long as the coarser support dimensions are of the order of a few hundred meters. In such a scenario, the upscaled characteristics of the parameters are governed predominantly by the texture and structure of the soil in the domain. However, when upscaling fine-scale

hydraulic parameter data to much larger extents (hillslope scales and beyond), topography plays a bigger role and can no longer be ignored. Efforts to model hydrologic processes and phenomena, with particular emphasis on those occurring in the unsaturated zone, are conducted at various scales. We present here a study to isolate the influence of topographic variations on the effective, upscaled soil hydraulic parameters under different hillslope configurations. The power-averaging operator algorithm was used to aggregate fine-scale soil hydraulic parameters to coarser resolutions. Hydrologic scenarios were simulated using HYDRUS-3D for four different topographic configurations under different conditions to test the validity of the upscaled soil hydraulic parameters. The outputs from the simulations (fluxes and soil moisture states) were compared across multiple scales for validating the effectiveness of the upscaled soil hydraulic parameters. It was found that the power-averaging algorithm produced reasonably good estimates of effective soil hydraulic parameters at coarse scales. Further, a probable threshold dimension beyond which the topography dominates the soil hydraulic property variation was analyzed. On the basis of only the topography, the scaling algorithm was able to capture much of the variation in soil hydraulic parameters required to generate equivalent flows and soil moisture states in a coarsened domain.

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## Advances in mathematical modeling of turbulent transport in porous media

**Marcelo J.S. de Lemos**

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Design of efficient engineering equipment and environmental impact analyses can benefit from more accurate mathematical modeling of turbulent transport in porous media. Several engineering and natural systems can be seen as porous structures through which a working fluid permeates. Turbulence models proposed for such flows depend on the order of application of time and volume average operators. Two methodologies, following the two orders of integration, lead to different governing equations for the statistical quantities. This lecture reviews published models over the last decade, which were developed to characterize turbulent transport in porous media. The concept of *double-decomposition* is discussed and models are classified in terms of the order of application of time and volume averaging operators, among other peculiarities. For hybrid media, involving both a finite porous structure and a clear flow region, difficulties arise due to the proper mathematical treatment given at the interface. This lecture further presents and discusses numerical solutions for such hybrid medium. Recent extensions of the *double-decomposition* concept for treating reactive flow in inert porous media and transport in moving beds are also discussed.

## Singularity in the combustion wave profile for porous media with liquid fuels

### Alexei A. Mailybaev

Instituto Nacional de Matematica Pura e Aplicada (IMPA), Brazil

In this talk I describe mathematical singularities encountered in the study of in situ combustion in porous medium, when the fuel is a liquid of relatively low boiling temperature (for example, light oil). The resonance phenomenon is encountered in the structure of the combustion wave, where the local characteristic speed becomes equal to the wave speed. Physically, this point separates the vaporization region from the reaction/condensation region. Mathematically, the wave profile must pass through the saddle-point singularity on the folded surface in the configuration space. The role of this singularity for determining properties of the combustion wave is discussed.

## The wave curve method: numerical implementation

### D. Marchesin
Instituto Nacional de Matematica Pura e Aplicada (IMPA), Brazil

Scale invariant or time-asymptotic solutions for systems of conservation laws are given by sequences of rarefaction waves, shock waves and constant states, determined by the wave curve method. We demonstrate software that implements this method for system of several equations, possibly non-hyperbolic, possessing localized source or diffusive terms. One application is in solving injection problems in porous media for thermal, compositional multiphase flow.

## On porous media flows for three components and two phases

### Vitor M.M. Matos

Universidade do Porto, Portugal

We present a family of models for the flow of liquid and gaseous hydrocarbons in a porous medium, ranging from a simple one that is easily subjected to mathematical analysis to a complex one that is realistic. There are two phases but three hydrocarbons, one with a liquid with a very high boiling temperature (dead oil), a second one with an intermediate boiling temperature, and a third one that is gaseous.

# Combustion fronts in a porous medium with two parallel layers

## Jesus Carlos da Mota

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In the first part of the talk, we discuss a model for the lateral propagation of a combustion front through a porous medium with two parallel layers having different properties. The reaction involves oxygen and a solid fuel. In each layer, the model consists of a nonlinear reaction-diffusion-convection system, derived from balance equations and Darcy's law. Considering some simplifications we obtain a simple model whose variables are temperature and unburned fuel concentration in each layer. The model includes heat transfer between the layers. When the heat transfer coefficient is large, we can prove that combustion fronts as traveling waves exist for a very large range of parameters. For small heat transfer, combustion typically does not occur simultaneously in the two layers. The proofs use geometric singular perturbation theory.

In the second part of the talk, we discuss the existence of solution for the Cauchy problem associated to the model. For the particular case, where the fuel concentrations in both layers are known functions, the proofs are based on the monotone method of upper and lower solutions. For the general case, the proofs are strongly connected to fundamental solution proprieties of parabolic equations with variable coefficients.

## References

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# Coupled Hydro-Geomechanical Computational Models of Pre-Salt Reservoirs

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We propose a new iterative coupled formulation based on Biot's theory of poroelasticity for multiphase flows of linear compressible fluids in strongly heterogeneous carbonated rocks underneath saline formations displaying creep behavior with the viscous strain ruled by a nonlinear constitutive law of power-law type.

Within the framework of the so-called iteratively coupled methods [4,5] and fixed-stress split algorithm [6] we develop mixed finite element methods for the flow and geomechanics subsystems which furnish locally conservative Darcy velocity and total Lagrangian fluid mass content in the sense of Coussy [3], Such fields are input for the transport problem for the water saturation which is formulated in terms of the Lagrangian porosity. The numerical resolution of the saturation equation is accomplished within a fractional step method. The predictor step is discretized by a higher-order non-oscillatory finite volume central scheme whereas the corrector based on Godunov or Strang splittings. The geomechanics step is solved in larger domains including the over-burden, up to the surface, under-burden and side-burdens up to the far field where boundary conditions are enforced. Numerical simulations of a water-flooding problem in secondary oil recovery are presented in domains characterized by images provided by seismic data processing . In addition simulations including the nonlinear stress-strain behavior of the adjacent rocks are performed showing the effects of irreversible deformation upon finger grow and breakthrough curves.

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# *A spectral decomposition strategy for decoupling an integro-differential model of expansive porous media*

### *Saulo Pomponet Oliveira*

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This talk concerns one step of an iterative method for a non-local model of swelling porous media saturated by an electrolyte solution containing finite size ions [1]. This model leads to a system of equations constituted from a Poisson equation for electric double layer potential, and nonlinear Fredholm integral equations of second kind that describe the ion-particle correlation functions. The spectral decomposition of integral kernels provides a convenient basis for the collocation method, since the resulting collocation equations do not have integral terms, i.e., nonlinear Fredholm equations are decoupled into nonlinear algebraic equations and Fredholm integral eigenvalue problems. For the

discretization of the latter, we consider a Galerkin method with piecewise continuous bilinear functions and reduced integration with the trapezoidal rule, which is a low-order spectral element discretization of Fredholm integral eigenvalue problems [2].

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# *Predictive Modeling of Subsurface Flows*

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We are concerned with the development of a computationally fast and effective framework for the modeling and simulation of multiphase subsurface flows arising in multiscale heterogeneous porous media. Our strategy consists of combining:

- Model development and validation at the Darcy (or Lab) scale
- Development of accurate numerical methods and high-performance numerical simulators
- Pore scale modeling and simulation to generate relevant flow functions (relative permeability and capillary pressure functions)
- Uncertainty quantification methods for reservoir characterization and prediction of fluid flows

We will discuss the challenges/opportunities related to these research areas and then we focus on the new concept of multi-physics Markov chain Monte Carlo methods for reservoir characterization.  A set of problems of practical interest will be considered such as  $CO_2$ storage in deep saline aquifers, contaminant transport in the subsurface and oil/gas recovery.

# *Numerical solution of in-situ combustion model using finite element method combined with nonlinear complementarity algorithm*

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Due to the growing shortage of light oils and increasing global energy demand, the recovery of heavy oil becomes more attractive. This type of oil is viscous making difficult its recovery. One way to reduce the viscosity is applying thermal methods such as steam injection and in-situ combustion [1]. The application

of steam injection in offshore recovery, common in Brazil, is complicated from the practical point of view.

The model of in-situ combustion studied here consists of a system of nonlinear partial differential equations. These kind of problems present several analytical and numerical difficulties related, for example, to time evolution and a moving boundary. One possible way to control this problem is to rewrite the system as a complementarity problem [2].

Numerical simulations will be performed using the finite element method combined with Feasible Directions Algorithm for Nonlinear Complementarity Problems (FDA-NCP) at each time step. The finite element method allows to work with non-trivial geometries, general boundary conditions and nonlinear properties when it is compared to the finite difference approach. The solutions obtained using the proposed methodology will be validated using other numerical techniques and particular analytical solutions [3].

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## *Numerical Simulation of Reservoir-Wellbore Coupled Flow*

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Universidade Estadual de Campinas (UNICAMP)
*Grazione de Souza*
Universidade Estadual do Rio de Janeiro (UERJ)

This work focuses in a wellbore-reservoir flow coupling model applied to petroleum engineering. An inherent difficulty related to this problem is the multiscale nature of the flow problem: the computational cell grids which host the injection/producing wells are much larger than wellbore radius. As a consequence, average pressure in a cell is not a good approximation of the well pressure. Moreover, at the interface between well and reservoir the flow pattern changes from porous media to pipe flow.

We consider the 3D single-phase isothermal flow of a compressible fluid in porous media and 1D single-phase isothermal flow of a compressible fluid in the wellbore. In both regions, mass conservation, momentum balance and real gas equation of state are the governing equations. Conservative forms are used for continuity and momentum. Flow continuity and pressure equilibrium are considered at the interface between well and reservoir. The nonlinear partial differential equations can be rewritten in order to allow the coupling of the pressure field between the porous media (reservoir) and the free media (well) by means of an implicit formulation. A hybrid grid technique was applied to improve the description of the flow close to the wellbore. In addition, pressure in reservoir and wellbore are solved simultaneously,

while velocities in wellbore are updated at every time step. By means of a (local) linearization of the discretized equations an iterative procedure it designed for solving numerically the resulting linear system for the primitive variable pressure, which includes the wellbore/reservoir coupling into a new finite difference numerical formulation.

This methodology is useful for the analysis of transient pressure and velocity behavior in both reservoir and wellbore including the change in flow pattern from porous media to pipe flow. Effects of friction, convection and compressibility in wellbore hydraulics were successfully included in the numerical approach. For early times, storage effects are clearly captured by our new approach. Indeed, the numerical experiments of this work show to be in good agreement with those obtained from a commercial simulator.

## The Mathematical Theory of Three-Phase Porous Medium Flow

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### D. Marchesin

IMPA, Brazil

We provide an overview of the mathematical theory of scale-invariant waves occurring in three-phase flow in a porous medium. Included is a discussion of software for constructing solutions of initial-value problems.

## Using a Partially-Linear Koval's Type Model to Study the Flow of Supercritical $CO_2$, Water and Oil in Porous Media: Pressure-Driven and Buoyancy-Driven Flows

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### Dan Marchesin
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### Hans Bruining
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It is well known that there exist two different convective transport phenomena that take part in the motion of fluids within a porous medium: convection due to pressure gradients (pressure-driven flow) and buoyancy due to density differences between the fluids (buoyancy-driven flow). In this work we study

both phenomena and their interaction for a flow of Supercritical $CO_2$, water and oil in a reservoir. We use a Koval's type model in which the relative permeabilities of $CO_2$ and oil-rich phases are both linear functions depending solely on the saturation of the corresponding phase, while the relative permeability of the water-rich phase is a quadratic function which depends solely on the water's saturation. Our calculations describe the flow patterns reflecting the motion of the fluids. All the Riemann solutions are obtained in explicit form; they contain sequences of spreading waves, discontinuities, and homogeneous regions of fluids. The investigation uses the mathematical theory of conservation laws and computations.

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## Air injection rate effects in light oil recovery

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Air injection is a potential technique for oil recovery improvement. This process is based on a spontaneous combustion mechanism between the oil and the injected oxygen which can improve oil recovery through several physical mechanisms.

Several aspects of air injection process have been studied theoretically and experimentally in the past. According to previous studies, combustion of the light and medium viscosity oils is described by different mechanisms. In high temperature oxidation (HTO), the cracking occurs forming coke, which is subsequently oxidized at high temperatures. In low temperature oxidation (LTO), the oxygen is adsorbed or incorporated by the hydrocarbon molecules to form alcohols, aldehyde, acids or other oxygenated hydrocarbons. In medium temperature oxidation (MTO), the oxidation reaction leads to small reaction products such as water, CO or CO2.

This paper presents the numerical analysis of light oil recovery by in-situ combustion focusing on the effect of different air injection rates. We propose a theoretical model given by a system of multiphase flow equations with source terms describing reaction and vaporization rates, and an energy balance equation in a porous medium. For our simulations we describe how the combustion displaces the oil inside the porous media with five different (low to high) air injection rates. We conclude that the air injection rate changes significantly the combustion wave structure which may have important implications for the light oil recovery process.

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## Semi-analytical solution for microwave assisted water injection in porous media

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**_Pacelli L.J. Zitha_** (TU Delft, The Netherlands)

In the production of heavy oils, various methods have been studied both theoretically and experimentally. The focus of this work is on hot water or steam injection methods, where the water is heated by electro-magnetic (EM) waves and the injected fluids heat-up the oil in the reservoir to reduce its viscosity [3].

Recently the small-scale experiment was done, were water was injected into a porous medium and heated by electromagnetic waves (EM) at a frequency of 2.54 GHz, same as common microwave oven, [1].

We consider a simple mathematical model describing this experiment consisting of two balance laws for energy and water mass. We use the Duhamel's Principle and the theory of Conservation Laws to find a semi-analytical solution to this simple model [2]. The results are validated numerically using Finite Difference technique [4].

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## *Unusual thermal effects in Riemann solutions for two-phase flow in porous media*

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We consider a class of nonlinear systems of conservation laws arising in petroleum engineering that models the injection of a mixture of gas and oil, in any proportion, into a porous medium filled with a similar mixture. The two mixtures may have different temperatures. This class of models is useful in applications such as advanced recovery of gas or oil, and the cleanup of polluted sites. We present a particularly unusual feature found in this model – for an open set of Riemann data the solution is given by a single wave group, i.e., there is no intermediate constant state. The key aspect supporting this feature is the existence of structurally stable doubly sonic shock waves, which robustly connect slow rarefaction waves to fast rarefaction waves. We prove that, together, coincidence of characteristic speeds and a certain amount of genuine non-linearity are sufficient to trigger the aforementioned phenomenon.

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# Wave sequences for solid fuel adiabatic in-situ combustion in porous media

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We consider a simplified mathematical model for forward combustion due to injection of air into a porous medium containing solid fuel. We consider the Arrhenius law for the reaction rate. In the adiabatic case studied here, reaction at the back of the combustion zone ceases due to complete lack of fuel. Ahead of the combustion zone we consider that there is abundant fuel, under two distinct conditions surrounding the combustion front. The first condition is the oxygen controlled case, where there is a complete oxygen consumption. The second condition is the temperature-controlled case, where the temperature is considered to be the same as the initial reservoir temperature. For this second case we consider a cut-off in the Arrhenius law to simplify the mathematical analysis. As there is interaction between the combustion wave and other waves, we focus on the solution of the Riemann problem with combustion taking into account all possible waves. Given initial reservoir and injection conditions, we prove that there is a unique time asymptotic sequence of three waves for combustion with complete oxygen consumption while for the temperature-controlled case, there is a one parameter family of wave speeds and strengths.

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# How to predict soil water holding capacity using physically based equations?

### Wenceslau Teixeira

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The soil surface and porosity govern the capacity of soil to hold water. In soil science, many functions were developed to estimate parameters of equations to describe soil water retention and soil hydraulic properties. These functions were created using statistical techniques based in soil properties parameters less laborious and cheaply obtained than the direct measurements of soil hydraulic parameters. These functions are called *pedotransfer functions (PTF)* and fill the gap between the available soil data and the properties required for modeling soil water fluxes. The physically based equations approach to obtain functions of water retention and hydraulic conductivity of the porous media (soils) may be advantageous, as the predictors will not be select by statistical procedures and be strongly influenced by the available

data base and technique used, moreover the predictors will have also a physical meaning. Examples of physically based equations to predict soil water content in function of dielectric conductivity in soils is given and the demand of physically based equation to predict soil water hold capacity and soil hydraulic conductivity is addressed.

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# Investigation for the Recovery of Medium-Heavy Oil by Gas Injection

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Enhanced oil recovery by $CO_2$ injection is becoming increasingly important means to improve the recovery of (medium-heavy) oil reservoir. In this paper will discuss core-flood tests done to investigate the potential of several $CO_2$ and $N_2$ injection schemes. $CO_2$, $N_2$, and brine were injected either (1) on continuous mode or (2) in a water-alternating-gas (WAG) mode into Bentheimer sandstone cores previously saturated with an heavy oil, having a viscosity in the range of 700 to 1,000 mPa·s and a density ranging from 16 to 17 °API. The injection strategies tested resulted in substantial incremental oil recovery (relative to OIIP), but values hereof varied from strategy to strategy. The core-flood test with continuous $CO_2$ injection following extensive water flooding resulted in an incremental recovery of 31.1 % over tertiary $CO_2$ flooding stage. Continuous $N_2$ and $CO_2$ injection in a secondary mode resulted in incremental recoveries of respectively 14.8 % and 24.5 % over secondary drainage stage. Subsequent long water slugs resulted in both cases in further increase of the recovery by 48.1 % and 29.2 %. $N_2$ and $CO_2$ WAG with WAG ratios and slug sizes chosen based on optimized reservoir simulations resulted in similar recovery factors. Overall recoveries for five schemes used in core-floods were in the range of 49.6 to 65.9 %. The paper discusses the possible mechanisms responsible for the oil displacement for each injection strategy and discusses conceptual ideas for mathematical and numerical modeling.

# *List of Speakers*

Eduardo Abreu (Department of Applied Mathematics-IMECC/UNICAMP, Campinas)

Amaury C. Alvarez (Oceanology Institute, C. Habana, Cuba)

Hans Bruining (Delft University of Technology, The Netherlands)

Grigori Chapiro (Dep. Matemática, UFJF, Juiz de Fora)

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