

Complete twisted cubics

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The predictions that Schubert did on twisted cubics in his book “Kalkül der abzählenden Geometrie” in 1879 are still very far from being understood. Many compactifications of the space of twisted cubics have appeared since then, and many questions have been answered, but nobody has gotten any close to the extremely rich and symmetric structure Schubert had in mind - nowadays, we would say he was thinking about 11 boundary divisors, while the more common compactifications, the Hilbert scheme and the space of stable maps, have only 2. In this talk, we will try to take a step in this direction; instead of compactifying the (homogeneous) space of twisted cubics PGL_4/PGL_2 as it is, we will compactify PGL_4 first, into the so-called space of *complete collineations*, and then take the GIT quotient by PGL_2 . The space so obtained is very symmetric; in fact, following the theory of homogeneous spaces, it is possible to link plenty of geometric properties of this space to representation theoretic properties of PGL_4 and PGL_2 . In this way, intersection theory on this space becomes just a combinatorial problem involving generating functions, partition functions, and interpolation; the number 56960 of twisted cubics tangent to 12 given planes is just the integral of a piecewise polynomial over a 3 dimensional region; the 11 degenerations that Schubert had in mind are just equivariant valuations on a weight lattice. This is a work in progress towards my PhD thesis.